ABSTRACT

Similarity search in database systems is becoming an increasingly important task in modern application domains such as multimedia, molecular biology, medical imaging, computer aided engineering, marketing and purchasing assistance as well as many others. In this paper, we show how visualizing the hierarchical clustering structure of a database of objects can aid the user in his time consuming task to find similar objects. We present related work and explain its shortcomings which led to the development of our new methods. Based on reachability plots, we introduce approaches which automatically extract the significant clusters in a hierarchical cluster representation along with suitable cluster representatives. These techniques can be used as a basis for visual data mining. We implemented our algorithms resulting in an industrial prototype which we used for the experimental evaluation. This evaluation is based on a real world test data set and points out that our new approaches to automatic cluster recognition and extraction of cluster representatives create meaningful and useful results.

Categories and Subject Descriptors
H.2.8 [Database Management]: Database Applications – data mining, image databases, spatial databases and GIS.

General Terms
Algorithms

Keywords
similarity search, visual data mining, clustering

1. INTRODUCTION

In the last ten years, an increasing number of database applications has emerged for which efficient and effective support for similarity search is substantial. The importance of similarity search grows in application areas such as multimedia, medical imaging, molecular biology, computer aided engineering, marketing and purchasing assistance, etc. [9, 1, 7, 8, 2, 5, 6, 10]. Particularly, the task of finding similar shapes in 2-D and 3-D becomes more and more important. Examples for new applications that require the retrieval of similar 3-D objects include databases for molecular biology, medical imaging and computer aided design.

Hierarchical clustering was shown to be effective for evaluating similarity models [12, 11]. Especially, the reachability plot generated by OPTICS [4] is suitable for assessing the quality of a similarity model. Furthermore, visually analyzing cluster hierarchies helps the user, e.g., an engineer, to find and group similar objects. Solid cluster extraction and meaningful cluster representatives form the foundation for providing the user with significant and quick information.

In this paper, we introduce algorithms for automatically detecting hierarchical clusters along with their corresponding representatives. In order to evaluate our ideas, we developed a prototype called BOSS (Browsing OPTICS Plots for Similarity Search). BOSS is based on techniques related to visual data mining. It helps to visually analyze cluster hierarchies by providing meaningful cluster representatives.

The remainder of the paper is organized as follows: After briefly introducing reachability plots, we present in Section 2 the application areas of hierarchical clustering along with the corresponding requirements in the industrial and in the scientific community which motivated the development of BOSS. In Sections 3 and 4, we introduce the notions of cluster recognition and cluster representatives respectively, which form the theoretical foundations of BOSS. In Section 5, we describe the actual industrial prototype we developed and evaluate its usefulness. The paper concludes in Section 6 with a short summary and a few remarks on future work.

2. HIERARCHICAL CLUSTERING

In this section, we outline the application ranges which led to the development of our interactive browsing tool, called BOSS. In order to understand the connection between BOSS and the application requirements we first introduce the reachability plots computed by OPTICS, which served as a start-
The reachability plot generated by OPTICS can be cut at a higher reachability value. Objects having a small reachability value are closer and thus computed by OPTICS. Valleys in this plot indicate clusters: the reachability plot is much easier to analyse. The reachability distance of each database object to one of its predecessors in the database objects minimizing a binary relation called reachability which is in most cases equal to the minimum distance of each database object to one of its predecessors in the ordering. Instead of a dendrogram, which is the common representation of hierarchical clusterings, the resulting reachability plot is much easier to analyse. The reachability values can be plotted for each object of the cluster-ordering computed by OPTICS. Valleys in this plot indicate clusters: objects having a small reachability value are closer and thus more similar to their predecessor objects than objects having a higher reachability value.

The reachability plot generated by OPTICS can be cut at any level \( \varepsilon_{\text{cut}} \) parallel to the abscissa. It represents the density-based clusters according to the density threshold \( \varepsilon_{\text{cut}} \): A consecutive subsequence of objects having a smaller reachability value than \( \varepsilon_{\text{cut}} \) belongs to the same cluster. An example is presented in Figure 1: For a cut at the level \( \varepsilon_1 \) we find two clusters denoted as \( A \) and \( B \). Compared to this clustering, a cut at level \( \varepsilon_2 \) would yield three clusters. The cluster \( A \) is split into two smaller clusters denoted by \( A_1 \) and \( A_2 \) and cluster \( B \) decreased its size. Usually, for evaluation purposes, a good value for \( \varepsilon_{\text{cut}} \) would yield as many clusters as possible.

2.2 Application Ranges

BOSS was designed for three different purposes: visual data mining, similarity search and evaluation of similarity models. For the first two applications, the choice of the representative objects of a cluster is the key step. It helps the user to get a meaningful and quick overview over a large existing data set. Furthermore, BOSS helps scientists to evaluate new similarity models.

2.2.1 Visual Data Mining

As defined in [3], visual data mining is a step in the KDD process that utilizes visualization as a communication channel between the computer and the user to produce novel and interpretable patterns. Based on the balance and sequence of the automatic and the interactive (visual) part of the KDD process, three classes of visual data mining can be identified.

- Visualization of the data mining result:
  - An algorithm extracts patterns from the data. These patterns are visualized to make them interpretable. Based on the visualization, the user may want to return to the data mining algorithm and run it again with different input parameters (cf. Figure 2a).

- Visualization of an intermediate result:
  - An algorithm performs an analysis of the data not producing the final patterns but an intermediate result which can be visualized. Then the user retrieves the interesting patterns in the visualization of the intermediate result (cf. Figure 2b).

- Visualization of the data:
  - Data is visualized immediately without running a sophisticated algorithm before. Patterns are obtained by the user by exploring the visualized data (cf. Figure 2c).

The approach presented in this paper belongs to the second class. A hierarchical clustering algorithm is applied to the data, which extracts the clustering structure as an intermediate result. There is no meaning associated with the generated clusters. However, our approach allows the user to visually analyze the contents of the clusters. The clustering algorithm used in the algorithmic part is independent from an application. It performs the core part of the data mining process and its result serves as a multi-purpose basis for further analysis directed by the user. This way the user may obtain novel information which was not even known to exist in the data set. This is in contrast to similarity search where the user is restricted to find similar parts respective to a query object and a predetermined similarity measure.

2.2.2 Similarity Search

The development, design, manufacturing and maintenance of modern engineering products is a very expensive and complex task. Effective similarity models are required for two- and three-dimensional CAD applications to cope with rapidly growing amounts of data. Shorter product cycles and a greater diversity of models are becoming decisive competitive factors in the hard-fought automobile and aircraft market. These demands can only be met if the engineers have an overview of already existing CAD parts. It would be desirable to have an interactive data browsing tool which depicts the reachability plot computed by OPTICS in a user
friendly way together with appropriate representatives of the clusters. This clear illustration would support the user in his time-consuming task to find similar parts. From the industrial user’s point of view, this browsing tool should meet the following two requirements:

- The hierarchical clustering structure of the dataset is revealed at a glance. The reachability plot is an intuitive visualization of the clustering hierarchy which helps to assign each object to its corresponding cluster or to noise. Furthermore, the hierarchical representation of the clusters using the reachability plot helps the user to get a quick overview over all clusters and their relation to each other. As each entry in the reachability plot is assigned to one object, we can easily illustrate some representatives of the clusters belonging to the current density threshold $\varepsilon_{cut}$ (cf. Figure 3).

- The user is not only interested in the shape and the number of the clusters, but also in the specific parts building up a cluster. As for large clusters it is rather difficult to depict all objects, representatives of each cluster should be displayed. To follow up a first idea, these representatives could be simply constructed by superimposing all parts belonging to the regarded cluster (cf. Figure 4). We can browse through the hierarchy of the representatives in the same way as through the OPTICS plots.

This way, the cost of developing and producing new parts could be reduced by maximizing the reuse of existing parts, because the user can browse through the hierarchical structure of the clusters in a top-down way. Thus the engineers get an overview of already existing parts and are able to navigate their way through the diversity of existing variants of products, such as cars.

### 2.2.3 Evaluation of Similarity Models

In general, similarity models can be evaluated by computing $k$-nearest neighbour queries ($k$-nn queries). As shown in [12], this evaluation approach is subjective and error-prone because the quality measure of the similarity model depends on the results of a few similarity queries and, therefore, on the choice of the query objects. A model may perfectly reflect the intuitive similarity according to the chosen query objects and would be evaluated as “good” although it produces disastrous results for other query objects.

A better way to evaluate and compare several similarity models is to apply a clustering algorithm. Clustering groups a set of objects into classes where objects within one class are similar and objects of different classes are dissimilar to each other. The result can be used to evaluate which model is best suited for which kind of objects. It is more objective since each object of the data set is taken into account to evaluate the data models.

### 3. CLUSTER RECOGNITION

In this section, we address the first task of automatically extracting clusters from the reachability plots. After a brief discussion of recent work in that area, we propose a new approach for hierarchical cluster recognition based on reachability plots.

#### 3.1 Recent Work

To the best of our knowledge, there are only two methods for automatic cluster extraction from hierarchical represen-
tations such as reachability plots or dendrograms — both are also based on reachability plots. Since clusters are represented as valleys (or dents) in the reachability plot, the task of automatic cluster extraction is to identify significant valleys.

The first approach proposed in [4] called $\xi$-clustering is based on the steepness of the valleys in the reachability plot. The steepness is defined by means of an input parameter $\xi$. The method suffers from the fact that this input parameter is difficult to understand and hard to determine. Rather small variations of the value $\xi$ often lead to drastic changes of the resulting clustering hierarchy. As a consequence, this method is unsuitable for our purpose of automatic cluster extraction.

The second approach was proposed recently by Sander et al. [14]. The authors describe an algorithm called cluster_tree that automatically extracts a hierarchical clustering from a reachability plot and computes a cluster tree. It is based on the idea that significant local maxima in the reachability plot separate clusters. Two parameters are introduced to decide whether a local maximum is significant: The first parameter specifies the minimum cluster size, i.e. how many objects must be located between two significant local maxima. The second parameter specifies the ratio between the reachability of a significant local maximum $m$ and the average reachabilities of the regions to the left and to the right of $m$. The authors in [14] propose to set the minimum cluster size to 0.5% of the data set size and the second parameter to 0.75. They empirically show, that this default setting approximately represents the requirements of a typical user.

Although the second method is rather suitable for automatic cluster extraction from reachability plots, it has one major drawback. Many real-world data sets consist of narrowing clusters, i.e. clusters each consisting of exactly one smaller sub-cluster (which may also be a narrowing cluster).

Since the algorithm cluster_tree runs through a list of all local maxima (sorted in descending order of reachability) and decides at each local maximum $m$, whether $m$ is significant to split the objects to the left of $m$ and to the right of $m$ into two clusters, the algorithm cannot detect such narrowing clusters. These clusters cannot be split by a significant maximum. Figure 5 illustrates this fact. The narrowing cluster $A$ consists of one cluster $B$ which is itself narrowing consisting of one cluster $C$ (the clusters are indicated by dashed lines). The algorithm cluster_tree will only find cluster $A$ since there are no local maxima to split clusters $B$ and $C$.

### 3.2 Drop-Down Clustering

Our new approach for automatic extraction of clusters from a reachability plot has some affinity to [14]. We also require a minimum cluster size and, although we do not require this explicitly, can install a ratio between the reachabilities at the boundary of a cluster and inside a cluster.

Since our method works in a top-down fashion, we call it Drop-Down Clustering. The idea behind is the successive use of the visual interpretation of the cluster ordering — as described in Figure 1 — which is based on the fact that the reachability plot can be cut by any level $\varepsilon_{cut}$ to the abscissa to extract a clustering. Starting from an initial clustering we simply drop the $\varepsilon_{cut}$-value in order to find substructures. Since it is not practical to test each possible $\varepsilon_{cut}$-value, we have to extract interesting values for a cut from the reachability values of the objects.

The Drop-Down Clustering algorithm starts by generating an initial root clustering. This does not contain all elements of the plot, as clusters separated by noise are assumed to be not related. Basically, a set of clusters forms the basis for a set of hierarchical clusters. This initial clustering is generated as follows: The objects in the reachability plot are sorted by descending reachability distance while retaining relative order among equal elements. The sorted list is now scanned until two objects are found whose indices are more than $MinPts$ apart, indicating that every element in between these two is smaller than either, thus constituting a dip in the graph. A top level cluster has been found, and all elements included in this cluster are removed from the sorted list. The scan can now continue until all elements have been removed or viewed.

The second part of the algorithm now separately analyzes each cluster found during the initial clustering. The extraction of further (sub-)clusters is a recursive procedure. The procedure starts with a set of elements from the reachability graph which is sorted by descending reachability values where elements having the same reachability value are arranged according to the cluster ordering. Figure 6 shows a sample cluster and its associated sorted element list. The first two elements of this sorted list are the edges containing all other elements (shown by the elements denoted by “m” in Figure 6). Starting at the third element, this list is sequentially tested for an object not adjacent to an edge. If an object is adjacent to an edge, the adjoining edge is moved to this object, shrinking the “pool” in the reachability graph. Should an object not be adjacent to either edge, then it must be inside the “pool” indicated by a local “bump” in the reachability graph (for instance the element denoted by the “+” in the sorted list in Figure 6). Three cases can be distinguished:

1. There exists a subcluster extending from the left edge to the bump
2. There exists a subcluster from the bump to a succeeding element of the same height
3. There exists a subcluster from the bump to the right edge.

Should a subcluster be found, it may be added to the resulting cluster hierarchy, after which it is then recursively processed to discover potential substructures. All discovered subclusters must conform to the following constraints:

1. The minimum cluster size constraint of $\text{MinPts}$ objects must be satisfied, i.e. there are at least $\text{MinPts}$ objects located between the start point and the end point of the cluster (e.g. Cluster $B$ in Fig. 5 must contain at least $\text{MinPts}$ objects).

2. The current cluster has at least $\text{MinPts}$ objects less than the cluster of its parent node in the hierarchy (e.g. Cluster $B$ in Fig. 5 must have at least $\text{MinPts}$ objects less than cluster $A$).

Let us note, that we could also claim a minimum ratio of reachabilities at the boundary of a cluster and inside a cluster as postulated in [14] or increment/decrement the required minimum cluster size.

Obviously, the Drop-Down algorithm is able to extract narrowing clusters. First experimental comparisons with the methods in [14] and [4] are presented in Section 5.

4. CLUSTER REPRESENTATIVES

In this section, we present three different approaches to determine representative objects for clusters computed by OPTICS. In the following, we assume that $DB$ is a database of multimedia objects, $\text{dist} : DB \times DB \rightarrow \mathbb{R}$ is a metric distance function on objects in $DB$ and $N_e(o) := \{ q \in DB \mid \text{dist}(o, q) \leq \varepsilon \}$ where $o \in DB$ and $\varepsilon \in \mathbb{R}$. A cluster $C \subseteq DB$ is represented by a set of $k$ objects of the cluster ($k$ should be a user defined integer), denoted as $\text{Rep}(C)$. We want to point out, that a representative must be a real object of the data set. A simple approach could be to superimpose all objects of a cluster to build the representative as it is depicted in Figure 4. However, this approach has the huge drawback that it is limited to image data and the representatives on a higher level of the cluster hierarchy will be rather unclear.

4.1 Extensions of the Medoid-Approach

Many partitioning clustering algorithms are known to use medoids as cluster representatives. The medoid of a cluster $C$ is the closest object to the mean of all objects in $C$. The mean of $C$ is also called centroid. For $k > 1$ we could choose the $k$ closest objects to the centroid of $C$ as representatives.

The choice of medoids as cluster representative is somehow questionable. Obviously, if $C$ is not of convex shape, the medoid is not really meaningful.

An extension of this approach coping with the problems of clusters with non-convex shape is the computation of $k$ medoids by applying a $k$-medoid clustering algorithm to the objects in $C$. The clustering using a $k$-medoid algorithm is rather efficient due to the expectation that the clusters are much smaller than the whole data set. This approach can also be easily extended to cluster hierarchies. At any level we can apply the $k$-medoid clustering algorithm to the merged set of objects from the child clusters or — due to performance reasons — merge the medoids of child clusters and apply $k$-medoid clustering on this merged set of medoids.

4.2 Minimizing the Core-Distance

The second approach to choose representative objects of hierarchical clusters uses the density-based clustering notion of OPTICS. To compute the reachability, OPTICS determines for each object the so-called core-distance:

Definition 1. (core-distance)

Let $o \in DB$, $\text{MinPts} \in \mathbb{N}$, $\varepsilon \in \mathbb{R}$, and $\text{MinPts-dist}(o)$ be the distance from $o$ to its $\text{MinPts}$-nearest neighbor. The core-distance of $o$ wrt. $\varepsilon$ and $\text{MinPts}$ is defined as follows:

$$\text{Core-Dist}(o) := \begin{cases} \infty & \text{if } |N_e(o)| < \text{MinPts} \\ \text{MinPts-dist}(o) & \text{otherwise.} \end{cases}$$

The core-distance of an object indicates the density of the surrounding region. The smaller the core-distance of an object $o$, the denser the region surrounding $o$. This observation led us to the choice of the object having the minimum core-distance ($\text{Core-Dist}(o)$) as representative of the respective cluster. Formally, $\text{Rep}(C)$ can be computed as:

$$\text{Rep}(C) := \{ o \in C \mid \forall x \in C : \text{Core-Dist}(o) \leq \text{Core-Dist}(x) \}.$$

We choose the $k$ objects with the minimum core-distances of the cluster as representatives.

The straightforward extension for cluster hierarchies is to choose the $k$ objects from the merged child clusters having the minimum core-distances.

4.3 Maximizing the Successors

Based on the core-distance, the reachability distance (or short: reachability) is defined as:

Definition 2. (reachability)

Let $o \in DB$, $\text{MinPts} \in \mathbb{N}$ and $\varepsilon \in \mathbb{R}$. The reachability of $o$ wrt. to $\varepsilon$ $\text{MinPts}$ relative to an object $p \in DB$ is defined as follows:

$$\text{Reach-Dist}(p, o) := \max (\text{Core-Dist}(p), \text{distance}(p, o))$$
The result of OPTICS is an ordering of the database minimizing the reachability relation. At each step of the ordering, the object \( o \) having the minimum reachability wrt. the already processed objects occurring before \( o \) in the ordering is chosen. Thus, if the reachability of object \( o \) is not \( \infty \), it is determined by \( \text{Reach-Dist}(p, o) \) where \( p \) is an object located before \( o \) in the cluster ordering. We call \( p \) the predecessor of \( o \).

**Definition 3.** (successors)

Let \( DB \) be a database of objects. For each object \( o \in DB \) in a cluster ordering computed by OPTICS, the set of successors is defined as \( S(o) := \{ s \in DB | o \text{ is the predecessor of } s \} \).

Let us note, that objects may have no predecessor, e.g. each object having a reachability of \( \infty \) does not have a predecessor, including the first object in the ordering. On the other hand, some objects may have more than one successor. In that case, some other objects have no successors.

The relationship of an object \( o \) to its successors \( S(o) \) is that the reachability of \( s \in S(o) \) is determined by \( \text{Reach-Dist}(o, s) \). We can model this relationship within each cluster as a directed successor graph where the nodes are the objects of one cluster and a directed edge from object \( o \) to \( s \) represents the relationship \( s \in S(o) \). Each edge \((x, y)\) can further be labeled by \( \text{Reach-Dist}(x, y) \). A sample successor graph is illustrated in Figure 7.

For the purpose of computing representatives of a cluster, the objects having many successors are interesting. Roughly speaking, these objects are responsible for the most density-connections within a cluster. The reachability values of these "connections" further indicate the distance between the objects.

Our third strategy selects the representatives of clusters by maximizing the number of successors and minimizing the according reachabilities. For this purpose, we compute for each object \( o \) of a cluster \( C \), having at least one successor, the Sum of the Invers Reachability distances of the successors of \( o \) within \( C \), denoted by \( \text{Sir}(o) \):

\[
\text{Sir}(o) := \begin{cases} 
0 & \text{if } S(o) = \emptyset \\
\sum_{s \in S(o)} \frac{1}{1 + \text{Reach-Dist}(o, s)} & \text{otherwise}
\end{cases}
\]

Since \( 1 + \text{Reach-Dist}(o, s) \geq 1 \) holds for the denominator, the impact of the number of successors is weighted over the significance of the reachability values.

Based on \( \text{Sir}(o) \), the representatives can be computed as follows:

\[
\text{Rep}(C) := \{ o \in C | \forall x \in C : \text{Sir}(o) \geq \text{Sir}(x) \}.
\]

If we want to select \( k \) representatives for \( C \) we simply have to choose the \( k \) objects with the maximum \( \text{Sir}(o) \) values.

5. **INDUSTRIAL PROTOTYPE AND EVALUATION**

5.1 **BOSS**

The development of the industrial prototype BOSS is a first step towards developing a comprehensive, scalable and distributed computing solution designed to make the efficiency of OPTICS and the analytical capabilities of BOSS available to a broader audience. Envisioned is a client/server system allowing users to provide their own data locally, along with an appropriate similarity model (cf. Figure 8).

The data provided by the user will be comprised of the objects to be clustered, as well as a data set to visualize these objects (e.g. VRML files for CAD data or JPEG images for multi-media data). Since this data resides on the user’s local computer and is not transmitted to the server there are only physical constraints on its size and type. In order for BOSS to be able to interpret this data, the user must supply his own similarity model with which the reachability data can be calculated.

The independence of the data processing and the data specification enables maximum flexibility. Further flexibility is introduced through the support of external visual representation. As long as the user is capable of displaying the visualization data in a browser, e.g. by means of a suitable plug-in, the browser will then load web pages generated by BOSS displaying the appropriate data. Thus, multimedia data such as images or VRML files can easily be displayed. By externalizing the visualization procedure we can resort to approved software components, which have been specifically developed for displaying objects which are of the same type as the objects within our clusters.
5.2 Evaluation

A main motivation for the development of BOSS has been the need to evaluate and compare different similarity models. This necessitates two steps, namely extracting a cluster structure, or cluster hierarchy structure, from which an interpretation of the data set can be attained with the help of representatives of the clustering results. In the following, three cluster recognition algorithms will vie among themselves, after which the three approaches for generating representatives will be evaluated on a single set of data (cf. Figure 9).

5.2.1 Cluster Recognition

Automatic cluster recognition is clearly very desirable when analyzing large sets of data. In this case, we will be looking at a subset of a database of CAD objects representing car parts. The results are depicted in Figure 9.

This data exhibits the commonly seen quality of unpronounced but nevertheless to the observer clearly visible clusters. The $\xi$-clustering approach (a) successfully recognizes a number of clusters while missing out on significant clusters.

The Drop-Down clustering has no trouble finding the intuitive cluster structure as indicated by the hierarchy at (b). The perhaps overzealous classification of concentric clusters means increased subsequent analysis, but aids in the automatic representation of recognized density patterns.

The $\text{cluster\_tree}$ algorithm (c) has difficulty with the overall flatness of the graph, as it needs a considerable height difference between crest and trough to produce concise results.

5.2.2 Cluster Representation

After a cluster recognition algorithm has analyzed the data, it is now possible to get a quick visual overview of the data. With the help of representatives, large sets of objects may be characterized through a single member of the set. We determine a sample cluster from the plot depicted in Figure 9 extracted by the Drop-Down algorithm to evaluate the different approaches for determining $k$ cluster representatives.

5.2.3 Summary

The results of our first experiments show that our new approaches for the automatic cluster extraction and for the determination of representative objects have the potential to outperform existing methods. It theoretically and empirically turned out, that our Drop-Down extraction algorithm seems to be more practical than recent work for automatic cluster extraction from hierarchical cluster representations. We also empirically showed that our approaches for the determination of cluster representatives is most likely more suitable than the simple (extended) medoid approach.

6. CONCLUSIONS

In this paper, we proposed hierarchical clustering combined with automatic cluster recognition and selection of representatives as a promising visualization technique. Its areas of application include visual data mining, similarity search and evaluation of similarity models. We surveyed three approaches for automatic extraction of clusters. The first method, $\xi$-clustering, fails to detect some clusters present in the clustering structure and suffers from the sensitivity concerning the choice of its input parameter. The algorithm $\text{cluster\_tree}$ is obviously unsuitable in the presence of narrowing clusters. We proposed a new method similar to $\text{cluster\_tree}$, called Drop-Down clustering. This algorithm is able to extract narrowing clusters and is evaluated
to deliver a good recognition of the intuitive clustering structure. Furthermore, we presented three different approaches to determine representative objects for clusters. The commonly known medoid approach is shown to be unsuitable for real-world data, while the approaches minimizing the core-distance and maximizing the successors both deliver good results. Finally, we described our industrial prototype, called BOSS, that implements the algorithms presented in this paper. It was used to evaluate the cluster recognition and representation methods.

In our future work, we plan to evaluate our algorithms with more data from various sources, utilizing the flexible framework which BOSS provides. Based on the results we may refine and extend the proposed methods in order to provide a useful visualization technique for industrial and scientific applications.

7. REFERENCES


