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Managing Massive Multiplayer Online Games

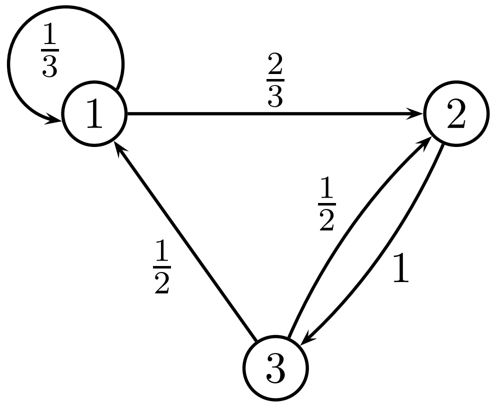
SS 2019

Exercise Sheet 8: Markov Chains

The assignments are due June 26, 2019

Assignment 8-1 *Markov Chains*

Given the Markov Chain M as depicted below. Nodes represent states, edges possible transitions and edge labels denote transition probabilities.



(a) Specify M in matrix notation. For that assume that the starting states are uniformly distributed and that sequences can only end after state 3 with a probability of 50%.

(b) What is the probability to observe the sequence $3-1-1-2-3$?

(c) What is the probability to observe the sequence $2-3-2-1-2$?

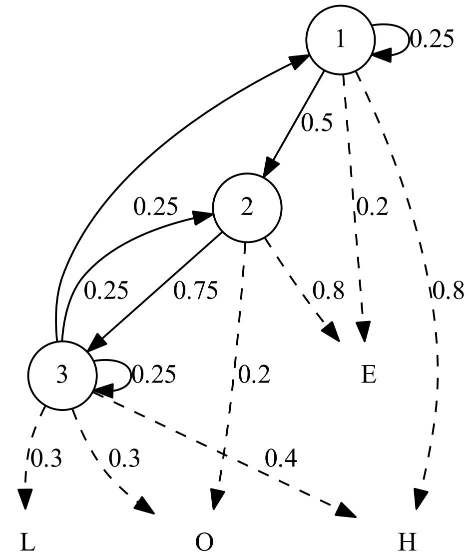
Assignment 8-2 *HMM: Calculation exercise*

Consider the Markov model given below.

(a) Specify the set of states A and the set of observations B. Deduce the transition matrix D and the output matrix F from the model. Assume that the starting probabilities are uniformly distributed and that the probabilities that sequences end in a state correspond to the values which are missing to the sum 1.

(b) Calculate the probability that the observation $O_1 = \{H, E, L, L, O\}$ is generated by the HMM.

(c) Which sequence $(s_1, s_2, \ldots, s_k)$ with $s_i \in A$ explains the observation $O_1 = \{H, E, L, L, O\}$ best?



Assignment 8-3 *HMM: Evaluation / Detection*

The Hidden Markov Model (HMM) $M = \{S, B, D, F\}$ with $S = \{A, B, C\}$, $B = \{\clubsuit, \heartsuit, \spadesuit\}$ is given as follows.

$$D = \begin{array}{c} \\ - \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{cccc} - & A & B & C \end{array} \\ \begin{pmatrix} 0 & 1/3 & 1/3 & 1/3 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 0 & 1/4 & 1/2 \\ 1/4 & 1/4 & 1/2 & 0 \end{pmatrix} \end{array} \qquad F = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} \clubsuit & \heartsuit & \spadesuit \end{array} \\ \begin{pmatrix} 1/4 & 3/4 & 0 \\ 0 & 0 & 1 \\ 0 & 1/4 & 3/4 \end{pmatrix} \end{array}$$

(a) Compute the probability of the observation $\clubsuit, \heartsuit, \spadesuit$ without algorithmic procedures. Tag the most probable sequence of states for the observation.

(b) Compute the probability of the observation $\clubsuit, \heartsuit, \spadesuit$ inductively by using the forward-variable

$$\alpha_j(1) = d_{-,j} f_{j,o_1} \quad \alpha_j(t+1) = \left(\sum_{i=1}^{|S|} \alpha_i(t) d_{i,j} \right) f_{j,o_{t+1}}$$

(c) Determine which sequence of states most probably produces the observation $\clubsuit, \heartsuit, \spadesuit$ by using the Viterbi-Algorithm.

$$\delta_j(1) = d_{-,j} f_{j,o_1} \quad \delta_j(t+1) = \left(\max_i \delta_i(t) d_{i,j} \right) f_{j,o_{t+1}}$$

$$\psi_j(1) = 0 \quad \psi_j(t+1) = \arg\max_i \delta_i(t) d_{i,j}$$