

Lehrstuhl für Datenbanksysteme und Data Mining  
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# Managing Massive Multiplayer Online Games

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On a player vs player competition ELO Ranking is used to rate the players. Assume  $\alpha = 0.07$  and  $\beta = 500$ . The following players and their ELO Scores are given:

- ▶ Player1: 1000
- ▶ Player2: 1200
- ▶ Player3: 800

During the competition the following outcomes occur in the given order:

- ▶ 1 beats 3
- ▶ 2 beats 1
- ▶ 3 beats 2



Let  $X_i \sim \mathcal{N}(\mu_i, \beta)$ ,  $X_j \sim \mathcal{N}(\mu_j, \beta)$ ,  $Y \sim \mathcal{N}(\mu_i - \mu_j, \beta)$  normal distributed variables.

$$\begin{aligned}P(X_i > X_j) &= P(X_i - X_j > 0) \\&= 1 - P(Y \leq 0) \\&= 1 - \Phi\left(\frac{0 - (\mu_i - \mu_j)}{\beta}\right) \\&= \Phi\left(\frac{\mu_i - \mu_j}{\beta}\right)\end{aligned}$$



Update-Rule:

$$\Delta = \alpha\beta\sqrt{\pi} \left( \frac{out + 1}{2} - \Phi \left( \frac{\mu_1 - \mu_2}{\sqrt{2}\beta} \right) \right)$$
$$\alpha = 0.07, \beta = 500$$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852	2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9992	0.9993
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000



$$\alpha = 0.07, \beta = 500$$

$$\begin{aligned}\Delta_{1\text{vs.}3} &= \alpha\beta\sqrt{\pi} \left( \frac{\text{out} + 1}{2} - \Phi \left( \frac{\mu_1 - \mu_3}{\sqrt{2}\beta} \right) \right) \\ &= \alpha\beta\sqrt{\pi} \left( \frac{1 + 1}{2} - \Phi \left( \frac{\mu_1 - \mu_3}{\sqrt{2}\beta} \right) \right) \\ &= \alpha\beta\sqrt{\pi} \left( \frac{-1 + 1}{2} - \Phi \left( \frac{-(\mu_1 - \mu_3)}{\sqrt{2}\beta} \right) \right) \\ &= 0.07 \cdot 500\sqrt{\pi} \left( 1 - \Phi \left( \frac{1000 - 800}{\sqrt{2} \cdot 500} \right) \right) \\ &= 35\sqrt{\pi} (1 - \Phi(0.282842712))\end{aligned}$$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852	2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Wolfram Alpha: 0.611351



$$\begin{aligned}\Delta_{1vs.3} &= 35\sqrt{\pi} (1 - \Phi(0.282842712)) \\ &= 35\sqrt{\pi} (1 - 0.611351) \\ &\approx 24.110184571\end{aligned}$$





$$\Delta_{1vs.3} \approx 24.110184571$$

$$\mu_1 \leftarrow \mu_1 + 24.110184571 = 1000 + 24.110184571 \approx 1024$$

$$\mu_3 \leftarrow \mu_3 - 24.110184571 = 800 - 24.110184571 \approx 776$$



$$\begin{aligned}\Delta_{2vs.1} &= \alpha\beta\sqrt{\pi} \left( \frac{out + 1}{2} - \Phi \left( \frac{\mu_2 - \mu_1}{\sqrt{2}\beta} \right) \right) \\ &= \alpha\beta\sqrt{\pi} \left( \frac{1 + 1}{2} - \Phi \left( \frac{\mu_2 - \mu_1}{\sqrt{2}\beta} \right) \right) \\ &= 0.07 \cdot 500\sqrt{\pi} \left( 1 - \Phi \left( \frac{1200 - 1024}{\sqrt{2} \cdot 500} \right) \right) \\ &= 35\sqrt{\pi} (1 - \Phi(0.248901587)) \\ &\approx 24.920931548\end{aligned}$$

$$\mu_2 \leftarrow \mu_2 + 24.920931548 = 1200 + 24.920931548 \approx 1225$$

$$\mu_1 \leftarrow \mu_1 - 24.920931548 = 1024 - 24.920931548 \approx 999$$



$$\begin{aligned}\Delta_{3vs.2} &= \alpha\beta\sqrt{\pi} \left( \frac{out + 1}{2} - \Phi \left( \frac{\mu_3 - \mu_2}{\sqrt{2}\beta} \right) \right) \\ &= \alpha\beta\sqrt{\pi} \left( \frac{1 + 1}{2} - \Phi \left( \frac{\mu_3 - \mu_2}{\sqrt{2}\beta} \right) \right) \\ &= 0.07 \cdot 500\sqrt{\pi} \left( 1 - \Phi \left( \frac{776 - 1225}{\sqrt{2} \cdot 500} \right) \right) \\ &= 35\sqrt{\pi} (1 - \Phi(-0.63498189)) \\ &\approx 45.737817106\end{aligned}$$

$$\mu_3 \leftarrow \mu_3 + 45.737817106 = 776 + 45.737817106 \approx 822$$

$$\mu_2 \leftarrow \mu_2 - 45.737817106 = 1225 - 45.737817106 \approx 1179$$



$$\mu_1 \approx 999$$

$$\mu_2 \approx 1179$$

$$\mu_3 \approx 822$$



Would the ELO Scores differ, if the order of the matches were 2 vs 3, 1 vs 3, 1 vs 2 but with the same results?



$$\begin{aligned}\Delta_{3vs.2} &= \alpha\beta\sqrt{\pi} \left( \frac{out + 1}{2} - \Phi \left( \frac{\mu_3 - \mu_2}{\sqrt{2}\beta} \right) \right) \\ &= \alpha\beta\sqrt{\pi} \left( \frac{1 + 1}{2} - \Phi \left( \frac{\mu_3 - \mu_2}{\sqrt{2}\beta} \right) \right) \\ &= 0.07 \cdot 500\sqrt{\pi} \left( 1 - \Phi \left( \frac{800 - 1200}{\sqrt{2} \cdot 500} \right) \right) \\ &= 35\sqrt{\pi} (1 - \Phi(-0.565685425)) \\ &\approx 44.305780742\end{aligned}$$

$$\mu_3 \leftarrow \mu_3 + 44.305780742 = 800 + 44.305780742 \approx 844$$

$$\mu_2 \leftarrow \mu_2 - 44.305780742 = 1200 - 44.305780742 \approx 1155$$



$$\begin{aligned}\Delta_{1vs.3} &= \alpha\beta\sqrt{\pi} \left( \frac{out + 1}{2} - \Phi \left( \frac{\mu_1 - \mu_3}{\sqrt{2}\beta} \right) \right) \\ &= \alpha\beta\sqrt{\pi} \left( \frac{1 + 1}{2} - \Phi \left( \frac{\mu_1 - \mu_3}{\sqrt{2}\beta} \right) \right) \\ &= 0.07 \cdot 500\sqrt{\pi} \left( 1 - \Phi \left( \frac{1000 - 844}{\sqrt{2} \cdot 500} \right) \right) \\ &= 35\sqrt{\pi} (1 - \Phi(0.220617316)) \\ &\approx 25.601899455\end{aligned}$$

$$\mu_1 \leftarrow \mu_1 + 25.601899455 = 1000 + 25.601899455 \approx 1026$$

$$\mu_3 \leftarrow \mu_3 - 25.601899455 = 844 - 25.601899455 \approx 818$$



$$\begin{aligned}\Delta_{2vs.1} &= \alpha\beta\sqrt{\pi} \left( \frac{out + 1}{2} - \Phi \left( \frac{\mu_1 - \mu_3}{\sqrt{2}\beta} \right) \right) \\ &= \alpha\beta\sqrt{\pi} \left( \frac{1 + 1}{2} - \Phi \left( \frac{\mu_1 - \mu_3}{\sqrt{2}\beta} \right) \right) \\ &= 0.07 \cdot 500\sqrt{\pi} \left( 1 - \Phi \left( \frac{1155 - 1026}{\sqrt{2} \cdot 500} \right) \right) \\ &= 35\sqrt{\pi} (1 - \Phi(0.18243355)) \\ &\approx 26.527847071\end{aligned}$$

$$\mu_2 \leftarrow \mu_1 + 26.527847071 = 1155 + 26.527847071 \approx 1182$$

$$\mu_1 \leftarrow \mu_3 - 26.527847071 = 1026 - 26.527847071 \approx 999$$





$$\mu_1 \approx 999$$

$$\mu_2 \approx 1182$$

$$\mu_3 \approx 818$$

Before:

$$\mu_1 \approx 999$$

$$\mu_2 \approx 1179$$

$$\mu_3 \approx 822$$