Ludwig-Maximilians-Universität München
Institut für Informatik
Prof. Dr. Matthias Schubert
Sebastian Schmoll

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Managing Massive Multiplayer Online Games

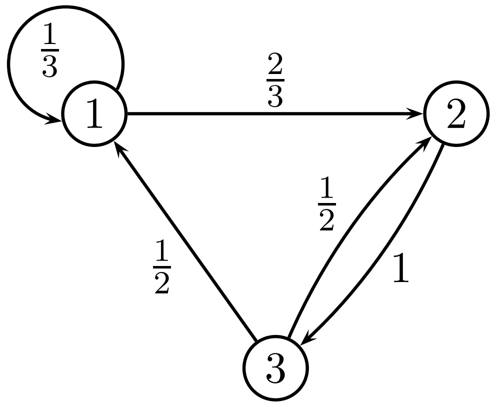
SoSe 2018

Exercise Sheet 9: Markov Chains

Discussion: June 13th, 2018

Exercise 9-1 *Markov Chains* (Homework)

The Markov Chain M is given below as a graph. Nodes represent states, edges possible transitions and edge labels transition probabilities.



(a) Specify M in matrix notation. For that assume that the starting states are uniformly distributed and that sequences can only end after state 3 with a probability of 50%.

(b) What is the probability to observe the sequence $3 - 1 - 1 - 2 - 3$?

(c) What is the probability to observe the sequence $2 - 3 - 2 - 1 - 2$?

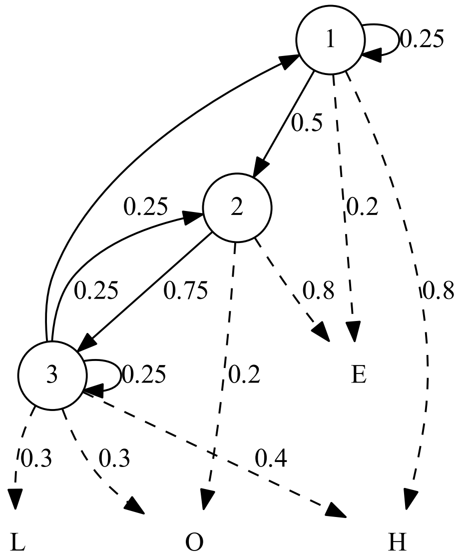
Exercise 9-2 *HMM: Calculation exercise* (Homework)

Regard the Markov model below.

(a) Specify the set of states A and the set of observations B. Deduce the transition matrix D and the output matrix F from the model. Assume that the starting probabilities are uniformly distributed and that the probability that sequences end in a state correspond to the values which are missing to the sum 1.

(b) Calculate the probability that the observation $O_1 = \{H, E, L, L, O\}$ is generated by the HMM.

(c) Which sequence $(s_1, s_2, \ldots, s_k)$ with $s_i \in A$ explains the observation $O_1 = \{H, E, L, L, O\}$ best?



Exercise 9-3 *HMM: Evaluation / Detection* **(Homework)**

The Hidden Markov Model (HMM) $M = \{S, B, D, F\}$ with $S = \{A, B, C\}$, $B = \{\clubsuit, \heartsuit, \spadesuit\}$ is given.

$$D = \begin{pmatrix} \times & - & A & B & C \\ - & 0 & 1/3 & 1/3 & 1/3 \\ A & 1/4 & 1/4 & 1/4 & 1/4 \\ B & 1/4 & 0 & 1/4 & 1/2 \\ C & 1/4 & 1/4 & 1/2 & 0 \end{pmatrix} \qquad F = \begin{pmatrix} \times & \clubsuit & \heartsuit & \spadesuit \\ A & 1/4 & 3/4 & 0 \\ B & 0 & 0 & 1 \\ C & 0 & 1/4 & 3/4 \end{pmatrix}$$

(a) Compute the probability of the observation $\clubsuit, \heartsuit, \spadesuit$ without algorithmic procedures. Tag the most probable sequence of states for the observation.

(b) Compute the probability of the observation $\clubsuit, \heartsuit, \spadesuit$ inductively with help of the forward-variable

$$\alpha_j(1) = d_{-,j} f_{j,o_1} \quad \alpha_j(t+1) = \left(\sum_{i=1}^{|A|} \alpha_i(t) d_{i,j} \right) f_{j,o_{t+1}}$$

(c) Determine with help of the Viterbi-Algorithm which sequence of states most probably produced the observation $\clubsuit, \heartsuit, \spadesuit$

$$\delta_j(1) = d_{-,j} f_{j,o_1} \quad \delta_j(t+1) = \left(\max_i \delta_i(t) d_{i,j} \right) f_{j,o_{t+1}}$$

$$\psi_j(1) = 0 \quad \psi_j(t+1) = \arg\max_i \delta_i(t) d_{i,j}$$