Exercise 12-1  Petri Nets

Explain if the following graphs are petri nets, workflow nets, or even sound workflow nets. Further express the graph as a sound workflow net if the graph is not yet a sound workflow net.

(a)

(b)

(c)

Exercise 12-2  α-Miner

(a) For the α-Miner algorithm, we use the relations $\succ, \rightarrow, ||, \#$ to denote direct successions, causality, parallelism or choice. Considering the set of activities $\{a, b, c\}$, give notion (graphically) about the following patterns and associate the right relations with them according to the activities having been used:

- Sequence Pattern
- XOR-Split and XOR-Join pattern
- AND-split and AND-join pattern
(b) Given the trace $L_1 = [(a, b, c, d), (a, c, b, d), (a, e, d)]$. Determine the following sets:

- Set of activities: $T_L = \{ t \in T | \exists \sigma \in L \ t \in \sigma \}$
- Set of start activities: $T_I = \{ t \in T | \exists \sigma \in L \ t = \text{first}(\sigma) \}$
- Set of end activities: $T_O = \{ t \in T | \exists \sigma \in L \ t = \text{last}(\sigma) \}$
- Set of paired activities:
  \[ X_L = \{(A, B) | A \subseteq T_L \land A \neq \emptyset \land B \subseteq T_L \land B \neq \emptyset \land \forall a \in A \forall b \in B a \rightarrow_L b \land \forall a_1, a_2 \in A a_1 \#_L a_2 \land \forall b_1, b_2 \in B b_1 \#_L b_2 \}\]
- Set of paired activities that are maximal:
  \[ Y_L = \{(A, B) \in X_L | \forall (A', B') \in X_L A \subseteq A' \land B \subseteq B' \implies (A, B) = (A', B') \}\]
- Set of places: $P_L = \{ p_{(A, B)} | (A, B) \in Y_L \} \cup \{ i_L, o_L \}$
- Flow relations:
  \[ F_L = \{ (a, p_{(A, B)}) | (A, B) \in Y_L \land a \in A \} \cup \{ p_{(A, B), b} | (A, B) \in Y_L \land b \in B \} \cup \{ (i_L, t) | t \in T_I \} \cup \{ (t, o_L) | t \in T_O \} \]
- Definition (no task): $\alpha$-Miner on event log $L$ is then defined as: $\alpha(L) = (P_L, T_L, F_L)$

(c) Construct the Footprint Table for trace $L_1$.  
