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# Knowledge Discovery and Data Mining I

WS 2018/19

## Exercise 8: Outlier Scores

### Exercise 8-1 Monotonicity of Simple Outlier Scores

Proof or give an counterexample for the following claims:

(a) If $o$ is an $D(\epsilon, \pi)$-outlier, it is also an $D(\epsilon', \pi)$-outlier for $\epsilon' \leq \epsilon$.

(b) If $o$ is an $D(\epsilon, \pi)$-outlier, it is also an $D(\epsilon, \pi')$-outlier for $\pi' \geq \pi$.

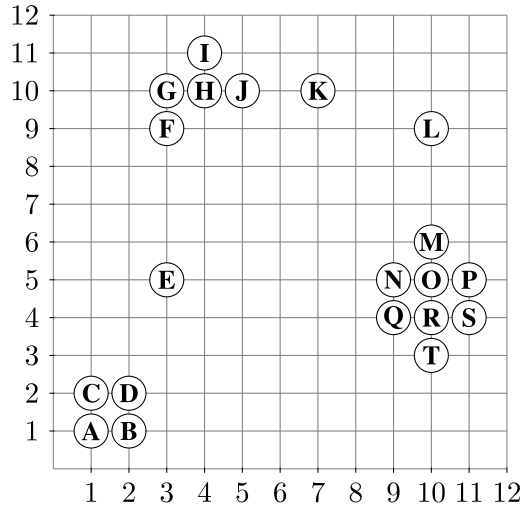
(c) If $o$ is an $k$NN-outlier for threshold $\tau$, it is also an $k'$NN-outlier for the same threshold with $k' > k$.

(d) If $o$ is an $k$NN-outlier for threshold $\tau$, it is also an $k$NN-outlier for threshold $\tau' < \tau$.

(e) The local density is monotonously decreasing in $k$, i.e. $ld_k(o) \geq ld_{k'}(o)$ for $k' > k$.

### Exercise 8-2 Outlier Scores

Given the following 2 dimensional data set:




As distance function, use Manhattan distance $L_1(a, b) := |a_1 - b_1| + |a_2 - b_2|$. The following table summarises the pairwise distances.

| | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | 0 | 1 | 1 | 2 | 6 | 10 | 11 | 12 | 13 | 13 | 15 | 17 | 14 | 12 | 13 | 14 | 11 | 12 | 13 | 11 |
| B | 1 | 0 | 2 | 1 | 5 | 9 | 10 | 11 | 12 | 12 | 14 | 16 | 13 | 11 | 12 | 13 | 10 | 11 | 12 | 10 |
| C | 1 | 2 | 0 | 1 | 5 | 9 | 10 | 11 | 12 | 12 | 14 | 16 | 13 | 11 | 12 | 13 | 10 | 11 | 12 | 10 |
| D | 2 | 1 | 1 | 0 | 4 | 8 | 9 | 10 | 11 | 11 | 13 | 15 | 12 | 10 | 11 | 12 | 9 | 10 | 11 | 9 |
| E | 6 | 5 | 5 | 4 | 0 | 4 | 5 | 6 | 7 | 7 | 9 | 11 | 8 | 6 | 7 | 8 | 7 | 8 | 9 | 9 |
| F | 10 | 9 | 9 | 8 | 4 | 0 | 1 | 2 | 3 | 3 | 5 | 7 | 10 | 10 | 11 | 12 | 11 | 12 | 13 | 13 |
| G | 11 | 10 | 10 | 9 | 5 | 1 | 0 | 1 | 2 | 2 | 4 | 8 | 11 | 11 | 12 | 13 | 12 | 13 | 14 | 14 |
| H | 12 | 11 | 11 | 10 | 6 | 2 | 1 | 0 | 1 | 1 | 3 | 7 | 10 | 10 | 11 | 12 | 11 | 12 | 13 | 13 |
| I | 13 | 12 | 12 | 11 | 7 | 3 | 2 | 1 | 0 | 2 | 4 | 8 | 11 | 11 | 12 | 13 | 12 | 13 | 14 | 14 |
| J | 13 | 12 | 12 | 11 | 7 | 3 | 2 | 1 | 2 | 0 | 2 | 6 | 9 | 9 | 10 | 11 | 10 | 11 | 12 | 12 |
| K | 15 | 14 | 14 | 13 | 9 | 5 | 4 | 3 | 4 | 2 | 0 | 4 | 7 | 7 | 8 | 9 | 8 | 9 | 10 | 10 |
| L | 17 | 16 | 16 | 15 | 11 | 7 | 8 | 7 | 8 | 6 | 4 | 0 | 3 | 5 | 4 | 5 | 6 | 5 | 6 | 6 |
| M | 14 | 13 | 13 | 12 | 8 | 10 | 11 | 10 | 11 | 9 | 7 | 3 | 0 | 2 | 1 | 2 | 3 | 2 | 3 | 3 |
| N | 12 | 11 | 11 | 10 | 6 | 10 | 11 | 10 | 11 | 9 | 7 | 5 | 2 | 0 | 1 | 2 | 1 | 2 | 3 | 3 |
| O | 13 | 12 | 12 | 11 | 7 | 11 | 12 | 11 | 12 | 10 | 8 | 4 | 1 | 1 | 0 | 1 | 2 | 1 | 2 | 2 |
| P | 14 | 13 | 13 | 12 | 8 | 12 | 13 | 12 | 13 | 11 | 9 | 5 | 2 | 2 | 1 | 0 | 3 | 2 | 1 | 3 |
| Q | 11 | 10 | 10 | 9 | 7 | 11 | 12 | 11 | 12 | 10 | 8 | 6 | 3 | 1 | 2 | 3 | 0 | 1 | 2 | 2 |
| R | 12 | 11 | 11 | 10 | 8 | 12 | 13 | 12 | 13 | 11 | 9 | 5 | 2 | 2 | 1 | 2 | 1 | 0 | 1 | 1 |
| S | 13 | 12 | 12 | 11 | 9 | 13 | 14 | 13 | 14 | 12 | 10 | 6 | 3 | 3 | 2 | 1 | 2 | 1 | 0 | 2 |
| T | 11 | 10 | 10 | 9 | 9 | 13 | 14 | 13 | 14 | 12 | 10 | 6 | 3 | 3 | 2 | 3 | 2 | 1 | 2 | 0 |

(a) Calculate the $D(\epsilon, \pi)$-outliers using

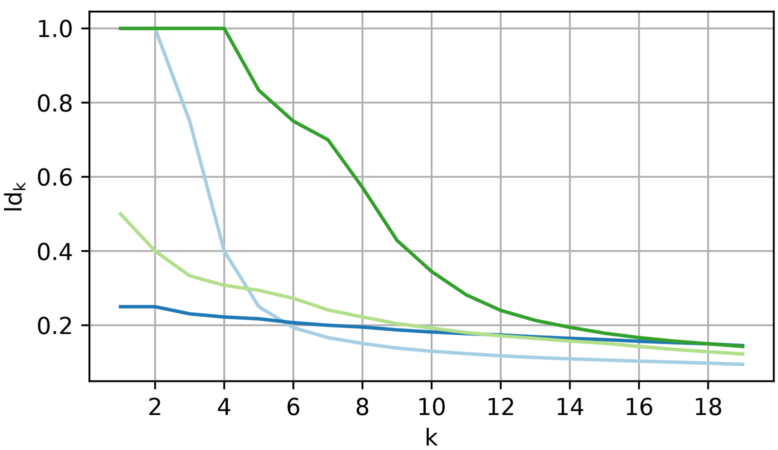
(i) $\epsilon = 2$ with $n\pi = 1$ and $n\pi = 2$.

(ii) $\epsilon = 4$ with $n\pi = 1$, $n\pi = 3$ and $n\pi = 4$.

(iii) $\epsilon = 6$ with $n\pi = 4$, $n\pi = 5$ and $n\pi = 6$.

(b) Calculate the $k$NN based outliers for $(k, \tau) = (3, 3)$ and $(k, \tau) = (5, 8)$. The point itself is counted as the 0-nearest neighbour.

(c) Given the following curves of the local density $ld_k$ for different values of $k$.




Can you identify which curve belongs to which point? Explain your mapping.