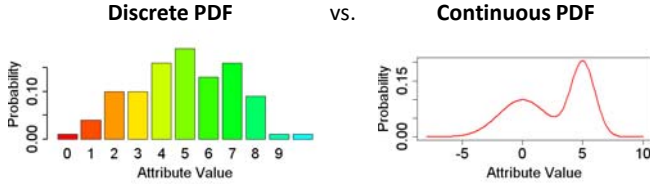
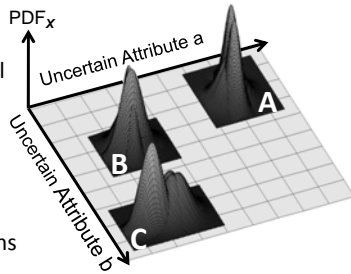


Uncertain Data Model:

- Uncertain attribute x : the value of x is given by a probabilistic density function (PDF), which describes all possible values v of x , associated with probability $P(x = v)$

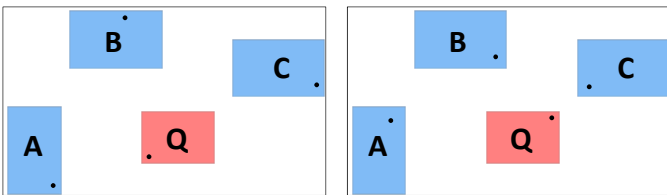


- An uncertain object X has at least $d \geq 1$ uncertain attributes
- Each uncertain attribute value of X is a random variable
- X is a random variable, where the set of attribute values of X is described by a multi-dimensional probability distribution
- X has a spatial region UR_x (Uncertain Region) where
 - $PDF_x(t) \geq 0$ if $t \in UR_x$ and
 - $PDF_x(t) = 0$ otherwise
- Uncertain object database: contains N uncertain objects



Problem Definition:

- Probabilistic k-nearest neighbor (PkNN) query: What are the k-nearest neighbors of the uncertain query object Q ?



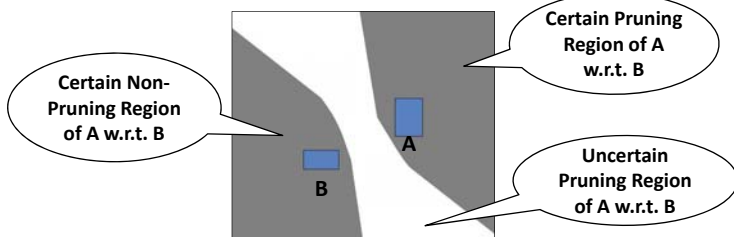
In some possible worlds, A is the nearest neighbor of Q ...
 ..., while in other possible worlds, that might not be the case.

General Framework:

- Approximation (Index):** Simplification of spatial-probabilistic keys
- Spatial Filter:** Filter objects according to simple spatial keys
- Probabilistic Filter:**
 - Derive lower/upper bounds of qualification probability (by means of simple spatial-probabilistic keys)
 - Filter objects according to lower/upper probability bounds
- Verification:**
 - Computation of the exact probability (very expensive)
 - Monte-Carlo Sampling (in general many samples needed)

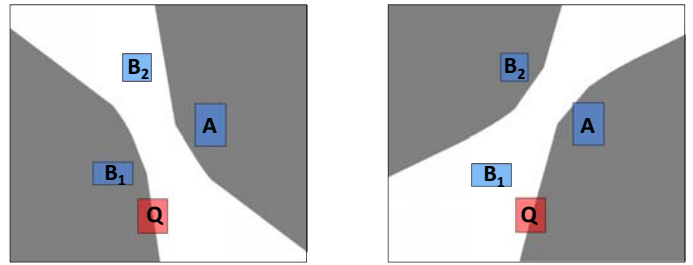
Spatial Filter:

- Pruning based on the spatial approximations only [1]:



Probabilistic Filter:

- Intuition for deriving pruning probability bounds:

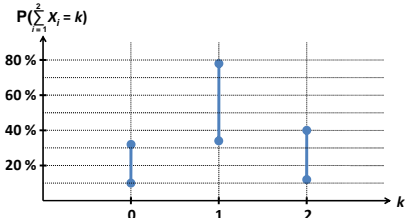


Lower Pruning Bound
 "B₁ is closer to Q than A with a Probability of at least x%"
 Upper Pruning Bound
 "B₂ is closer to Q than A with a Probability of at most x%"

Uncertain Generating Functions:

- Assume the following pruning bounds have been derived:
 - B₁ prunes A with a probability of at least 0.2 and at most 0.7
 - B₂ prunes A with a probability of at least 0.6 and at most 0.8
- What is the probability that at least (at most, exactly) k objects prune A?

- Given the Generating Function
 $(0.2x + 0.5y + 0.3) * (0.6x + 0.2y + 0.2)$



- Expansion yields:
 $0.12x^2 + 0.34x + 0.1 + 0.22xy + 0.16y + 0.06y^2$

- Formally let X_1, \dots, X_n be random predicates and let p_i^{lb} and p_i^{ub} be the respective lower and upper bounds of the probability that $X_i = true$

- Consider the expansion of the Uncertain Generating Function:

$$\prod_{i=1}^n p_i^{lb} x + (p_i^{ub} - p_i^{lb}) y + (1 - p_i^{ub})$$

- A lower bound of the probability that exactly k predicates are true is given by the coefficient of the term x^k
- An upper bound is given by the sum of all coefficients of the term $x^m y^n$ where $m \leq k$ and $m + n \geq k$

Problem of Dependencies:

- Approximate the probability that A is the NN of Q (assuming uniform distribution in Q):

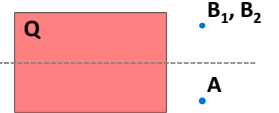
$$P(„B_1 \text{ is closer to Q than A“}) = 0.5$$

$$P(„B_2 \text{ is closer to Q than A“}) = 0.5$$

$$\Rightarrow P(„A \text{ is NN of Q“}) = 0.5 * 0.5 = 0.25$$

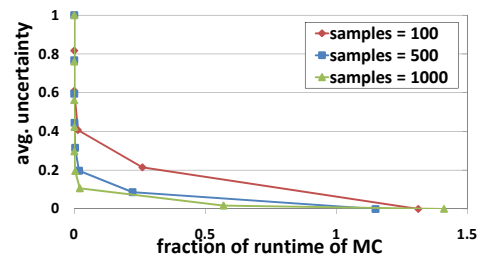
is WRONG!

$$\Rightarrow P_{lb} = 0 \text{ and } P_{ub} = 0.5 \text{ is CORRECT!}$$



Evaluation:

- Comparison with Monte-Carlo-based approach:



References:

- [1] T. Emrich, H.-P. Kriegel, P. Kröger, M. Renz, A. Züfle: Boosting Spatial Pruning: On Optimal Pruning of MBRs. SIGMOD Conference 2010, pp. 39-50.