Assignment 10-1  Path finding

The blue shapes represent obstacles in the field above. In the following we are looking for the shortest path the objects a), b), and c) have to follow to get past the obstacles and to the target.

Hint: To facilitate drawing, vertical and horizontal auxiliary lines are given which have a distance of one unit each.

(a) Draw the visibility graph for the point tagged with a) and determine the shortest path to the target.

(b) Draw the visibility graph for the circle tagged with b), which has a radius \( r = 1 \), and determine - if possible - the shortest path to the target.

(c) Draw the visibility graph for the triangle tagged with c) and determine - if possible - the shortest path to the target. Assume that the triangle is right and isosceles, with edge length 1.

For the solutions, please see the extra document provided on the website.

Assignment 10-2  Antagonistic Search

Consider an abstract game in which two players \( P_1 \) and \( P_2 \) are on turn alternately and can perform one of two actions every turn. At each instance of the game, the game situation \( GS \) for each player can be rated with a score function \( s(GS, P_i) \), with a higher score implying a better game situation.
In the following we want to decide which is the best possible action for player $P_1$, who has a turn. First $s(GS_0, P_1) = 0$ holds. The antagonistic search tree below shows a couple of game state scores and all possible actions of $P_1$ together with all corresponding reactions of $P_2$.

(a) Which nodes have to be examined if looking for the optimal strategy for $P_1$ with a MinMax-search and alpha-beta pruning?

(b) Does the order in which the nodes are visited make a difference?

For the solutions, please see the extra document provided on the website.

**Assignment 10-3  ELO Scores**

On a player vs player competition ELO Ranking is used to rate the players. Assume $\alpha = 0.07$ and $\beta = 500$. The following players and their ELO Scores are given:

- Player1: 1000
- Player2: 1200
- Player3: 800

During the competition the following outcomes occur in the given order:

- 1 beats 3
- 2 beats 1
- 3 beats 2

(a) Calculate the ELO Scores at the end of the competition.
We use the following update rule:

$$\Delta = \alpha \beta \left( \frac{\text{out} + 1}{2} - \Phi\left(\frac{\mu_1 - \mu_2}{\beta}\right)\right).$$

Note that this update rule is different from the one given in the script. Here, the constant factors are omitted. This corresponds to an easier (and modern?) version of the ELO score. Using this update rule, we get:

$$\Delta_{1v3} = 35(1 - \Phi\left(\frac{1000 - 800}{500}\right)) = 35(1 - 0.65542) = 12.0603$$
$$\mu_1 = 1012$$
$$\mu_3 = 788$$

$$\Delta_{2v1} = 35(1 - \Phi\left(\frac{1200 - 1012}{500}\right)) = 35(1 - 0.64803) = 12.31895$$
$$\mu_1 = 1000$$
$$\mu_2 = 1212$$

$$\Delta_{3v2} = 35(1 - \Phi\left(\frac{788 - 1212}{500}\right)) = 35(1 - (1 - 0.80234)) = 28.0819$$
$$\mu_3 = 816$$
$$\mu_2 = 1184$$

Also note that we only used $\text{out} = 1$ for calculating the updates (because we’ve always seen the game from the view of the winner). This is possible due to the updates being symmetric.

(b) Would the ELO Scores differ, if the order of the matches were 2 vs 3, 1 vs 3, 1 vs 2 but with the same results?

$$\Delta_{2v3} = 35(0 - \Phi\left(\frac{1200 - 800}{500}\right)) = 35(0 - \Phi(0.8)) = -27.585$$
$$\mu_2 = 1183$$
$$\mu_3 = 827$$

$$\Delta_{1v3} = 35(1 - \Phi\left(\frac{1000 - 827}{500}\right)) = 35(1 - \Phi(0.346)) = 12.71095$$
$$\mu_1 = 1012$$
$$\mu_3 = 815$$

$$\Delta_{1v2} = 35(0 - \Phi\left(\frac{1012 - 1173}{500}\right)) = 35(0 - \Phi(-0.322)) = -13.1068$$
$$\mu_1 = 999$$
$$\mu_2 = 1186$$

Now we’ve also calculated some updates from the perspective of the loser and thus used $\text{out} = -1$. However, due to symmetry, we again need to calculate the updates only once. We also can observe that the final ELO scores indeed differ if we change the order.