Assignment 2-1  Visibility

Consider an abstract game where agents move on a two-dimensional map. An agent can see objects and other agents within a circular region with radius \(s\), i.e., the sight distance.

(a) How to check if an agent \(p\) with position \((p.x, p.y)\) is able to see a circular object \(o\) with center \(o_m\) and radius \(o_r\)?
\[
\text{Dist}(p, o_m) \leq s + o_r
\]

(b) Let \(R\) be a rectangular approximation of a set \(S\) of agents. How to check if it is possible that an agent \(p \notin S\) can see agents in \(S\) without accessing the exact positions of the players in \(S\)?
\[
\text{MinDist}(p, S) \leq s
\]

(c) Let \(R_1\) and \(R_2\) be rectangular approximations of two disjoint sets \(S_1\) and \(S_2\) of agents. How to check if it is possible that there are pairs of agents \((p_1, p_2) \in S_1 \times S_2\) which can see one another without accessing the exact positions of the agents?
\[
\text{MinDist}(R_1, R_2) \leq s
\]

Hint: The following functions can be used:

- The Euclidean distance between two points \(p_1\) and \(p_2\):
\[
\text{Dist}(p_1, p_2) = \sqrt{(p_1.x - p_2.x)^2 + (p_1.y - p_2.y)^2}
\]

- The minimal Euclidean distance between a point \(p\) and a rectangle \(R\):
\[
\text{MinDist}(p, R) = \sum_{i=1}^{2} \left\{ \begin{array}{ll} |R^\text{min}_i - p_i|^2, & \text{if } R^\text{min}_i > p_i \\ |p_i - R^\text{max}_i|^2, & \text{if } p_i > R^\text{max}_i \\ 0, & \text{else} \end{array} \right. 
\]

- The minimal Euclidean distance between two rectangles \(A\) und \(B\):
\[
\text{MinDist}(A, B) = \sum_{i=1}^{2} \left\{ \begin{array}{ll} |A^\text{min}_i - B^\text{max}_i|^2, & \text{if } A^\text{min}_i > B^\text{max}_i \\ |B^\text{min}_i - A^\text{max}_i|^2, & \text{if } B^\text{min}_i > A^\text{max}_i \\ 0, & \text{else} \end{array} \right. 
\]

\(A_i\) is the projection of \(A\) onto dimension \(i\) and \(X^\text{min}\) (resp. \(X^\text{max}\)) is the minimum (resp. maximum) of an interval \(X\).
The map shown above is divided into a $4 \times 4$ equi-grid of micro zones. 32 objects are moving on the map with initial positions as shown above. The “Area of Interest” (AoI) of an object is circular with its radius corresponding to half of the side length of a micro zone. *(Note: they are shown for some objects in the Figure.)*

(a) Consider the object marked with *a*) in micro zone A4. To which micro zones has this object to subscribe initially?

A4 (because the AoI is only intersected by A4)

(b) From which objects does *a*) initially get positional information? Which objects initially get positional information from *a*)?

- *a*) gets positional information from all objects in A4 (because the AoI is only intersected by A4)
- *a*) sends positional information to all objects whose AoI is intersected by A4 (because *a*) registered/published in A4)

(c) To which micro zones must object *a*) subscribe if it moves as shown in the figure?

$A4 \rightarrow A3, A4, B3, B4 \rightarrow A3, A4, B3, B4$ (again, due to AoI-micro-zone-intersections)

(d) To which micro zones must object *b*) subscribe if it moves as shown in the figure?

$A1, A2, B1, B2 \rightarrow B2, B3, C2 \rightarrow A2, A3, B2, B3$
Index the dataset shown above by using

(a) a Quadtree with page size 2.

(b) a kD-Tree with page size 4. Begin by splitting the x-axis.

(c) an R-tree with page size 2. Build the R-tree with the sort-tile recursive algorithm.

The solutions for this assignment can be found in the other files that I’ve uploaded.