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Managing Massive Multiplayer Online Games

SS 2017

Exercise Sheet 8: Temporal Behavior

Discussion: June 28th, 2017

Exercise 8-1 *Suffix Trees*

The alphabet $A = \{A, B, C, D, N\}$ is given.

(a) Insert the sequence $G_1 = \{B, A, N, A, N, A\}$ into an empty suffix tree ST

(b) Additionally insert the sequence $G_2 = \{C, A, N, A, D, A\}$ into ST.

(c) Find the subsequence $S_1 = \{N, A, N, A\}$. Which sequence contains S_1?

(d) Which is the longest common subsequence of G_1 and G_2?

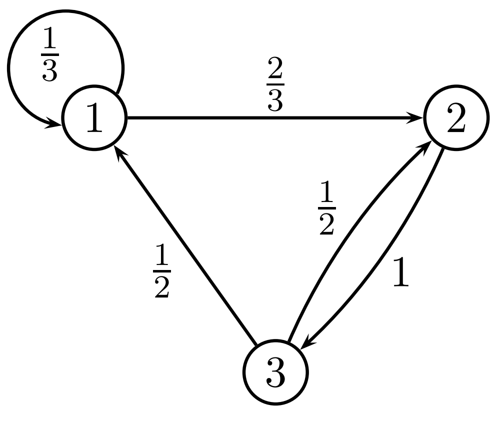
(e) Which extension would be necessary to support finding the most frequent subsequence of length n (or longer)?

Exercise 8-2 *Levenshtein Distance*

Compute the Levenshtein Distance between the sequences $BANANA$ and $CANADA$

Exercise 8-3 *Markov Chains*

The Markov Chain M is given below as a graph. Nodes represent states, edges possible transitions and edge labels transition probabilities.



(a) Specify M in matrix notation. For that assume that the starting states are uniformly distributed and that sequences can only end after state 3 with a probability of 50%.

(b) What is the probability to observe the sequence $3 - 1 - 1 - 2 - 3$?

(c) What is the probability to observe the sequence $2 - 3 - 2 - 1 - 2$?

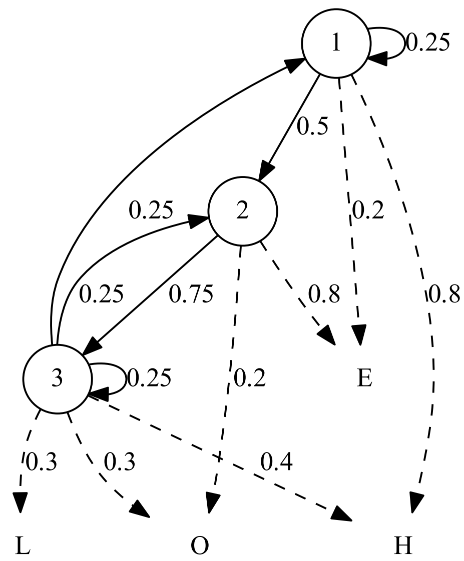
Exercise 8-4 *HMM: Calculation exercise*

Regard the Markov model below.

(a) Specify the set of states A and the set of observations B. Deduce the transition matrix D and the output matrix F from the model. Assume that the starting probabilities are uniformly distributed and that the probability that sequences end in a state correspond to the values which are missing to the sum 1.

(b) Calculate the probability that the observation $O_1 = \{H, E, L, L, O\}$ is generated by the HMM.

(c) Which sequence $(s_1, s_2, \ldots, s_k)$ with $s_i \in A$ explains the observation $O_2 = \{H, E, L, L, O\}$ best?



Exercise 8-5 *HMM: Evaluation / Detection*

The Hidden Markov Model (HMM) $M = \{S, B, D, F\}$ with $S = \{A, B, C\}$, $B = \{\clubsuit, \heartsuit, \spadesuit\}$ is given.

$$D = \begin{pmatrix} \times & - & A & B & C \\ - & 0 & 1/3 & 1/3 & 1/3 \\ A & 1/4 & 1/4 & 1/4 & 1/4 \\ B & 1/4 & 0 & 1/4 & 1/2 \\ C & 1/4 & 1/4 & 1/2 & 0 \end{pmatrix} \qquad F = \begin{pmatrix} \times & \clubsuit & \heartsuit & \spadesuit \\ A & 1/4 & 3/4 & 0 \\ B & 0 & 0 & 1 \\ C & 0 & 1/4 & 3/4 \end{pmatrix}$$

(a) Compute the probability of the observation $\clubsuit, \heartsuit, \spadesuit$ without algorithmic procedures. Tag the most probable sequence of states for the observation.

(b) Compute the probability of the observation ♣, ♡, ♠ inductively with help of the forward-variable

$$\alpha_j(1) = d_{-,j} f_{j,o_1} \quad \alpha_j(t+1) = \left(\sum_{i=1}^{|A|} \alpha_i(t) d_{i,j} \right) f_{j,o_{t+1}}$$

(c) Determine with help of the Viterbi-Algorithm which sequence of states most probably produced the observation ♣, ♡, ♠

$$\delta_j(1) = d_{-,j} f_{j,o_1} \quad \delta_j(t+1) = \left(\max_i \delta_i(t) d_{i,j} \right) f_{j,o_{t+1}}$$
$$\psi_j(1) = 0 \quad \psi_j(t+1) = \arg\max_i \delta_i(t) d_{i,j}$$