Exercise 5-1  
*Bot Detection with Bayes*

In the following regard an abstract game in which players regularly have to make decisions. Examples are:

- In which order to pick up objects
- In which direction the player should go at a crossroad inside of a labyrinth
- In which direction the player sends his exploring units

We assume that there are always four alternatives \{a_1, ..., a_4\} and that a BOT selects each of these with the same probability. With the help of log data it could be estimated empirically that real players select the alternatives with the following probabilities:

<table>
<thead>
<tr>
<th></th>
<th>a_1</th>
<th>a_2</th>
<th>a_3</th>
<th>a_4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>10%</td>
<td>20%</td>
<td>30%</td>
<td>40%</td>
</tr>
</tbody>
</table>

Player \( p_1 \) was observed to have the following sequence of decisions:

\[ O = [a_3, a_2, a_1, a_4, a_1, a_2, a_2, a_3, a_1] \]

In the following \( B \) is the event that player \( p_1 \) is a BOT and \( \overline{B} \) is the event that player \( p_1 \) is a real player.

(a) Calculate the probability \( P(O \mid B) \) that a BOT produces the above sequence.

(b) Calculate the probability \( P(O \mid \overline{B}) \) that a real player produces the above sequence.

(c) Assume that 1\% of all players are BOTs. Calculate the probability \( P(B \mid O) \) that player \( p_1 \) is a BOT.

Exercise 5-2  
*Probabilistic Balancing*

Regarding another game players can choose between several different settings (e.g. races, classes, fractions) in the beginning. Let \( s_1, ..., s_n \) be such settings.

Assume that 1000 games were monitored each between a player with settings \( s_1 \) and a player with settings \( s_2 \) (briefly \( s_1 \) vs \( s_2 \)). 400 of those were won by the player with settings \( s_1 \). Is this game fair concerning the settings \( s_1 \) and \( s_2 \)? Therefore calculate the possibility of this observation assuming that the game was fair, i.e. that the winning chances for both players were always 50\%. 