

**Managing Massive Multiplayer Online Games**  
 SS 2017

**Exercise Sheet 2: Space**

Discussion: May 10th, 2017

In the following regard an abstract game, where the players are in a two-dimensional world. In this world a player can perceive objects and other players inside a circular region with radius  $s_r$  (*sight distance*).

**Exercise 2-1**      *Visibility*

- (a) How can one efficiently check if a player  $p$  with position  $(p.x, p.y)$  is able to see a round object  $o$  with center  $o_m$  and radius  $o_r$ ?
- (b) Let  $R$  be a rectangular approximation of a set  $S$  of players. How can one efficiently check if it is possible that a player  $p \notin S$  can see players in  $S$  without accessing the exact positions of the players in  $S$ ?
- (c) Let  $R_1$  and  $R_2$  be rectangular approximations of two disjunct sets  $S_1$  and  $S_2$  of players. How can one efficiently check if it is possible that there are pairs of players  $(p_1, p_2) \in S_1 \times S_2$  which can see one another without accessing the exact positions of players in  $S_1$  and  $S_2$ ?

Hint: The following functions can be used:

- The Euclidean distance between two points  $p_1$  and  $p_2$ :

$$Dist(p_1, p_2) = \sqrt{(p_1.x - p_2.x)^2 + (p_1.y - p_2.y)^2}$$

- The minimal Euclidean distance between a point  $p$  and a rectangle  $R$ :

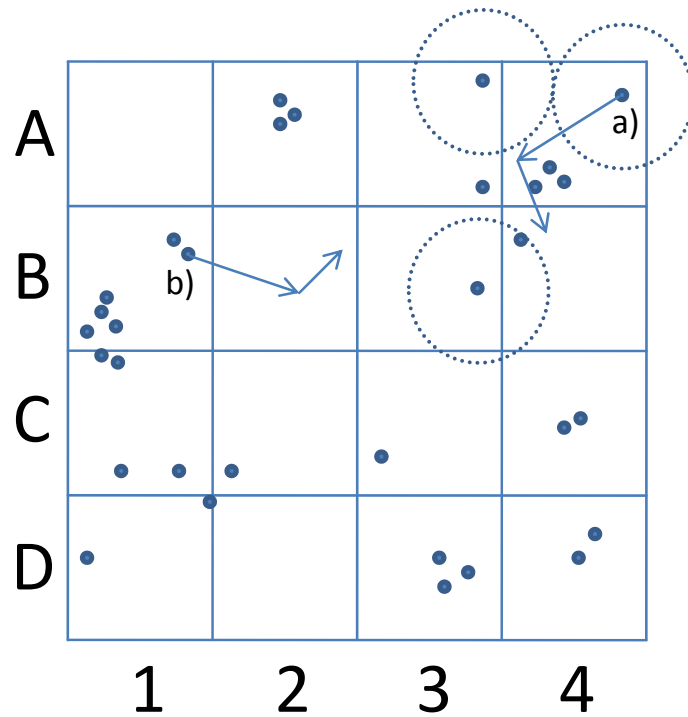
$$MinDist(p, R) = \sqrt{\sum_{i=1}^2 \begin{cases} |R_i^{min} - p_i|^2, & \text{if } R_i^{min} > p_i \\ |p_i - R_i^{max}|^2, & \text{if } p_i > R_i^{max} \\ 0, & \text{else} \end{cases}} \quad (1)$$

- The minimal Euclidean distance between two rectangles  $A$  and  $B$ :

$$MinDist(A, B) = \sqrt{\sum_{i=1}^2 \begin{cases} |A_i^{min} - B_i^{max}|^2, & \text{if } A_i^{min} > B_i^{max} \\ |B_i^{min} - A_i^{max}|^2, & \text{if } B_i^{min} > A_i^{max} \\ 0, & \text{else} \end{cases}} \quad (2)$$

Here  $A_i$  is the projection of  $A$  onto the dimension  $i$  (i.e. the  $i$ -th entry of a vector or the interval which describes the expansion of a rectangle in the  $i$ -th dimension) and  $X^{min}$  (resp.  $X^{max}$ ) is the minimum (resp. maximum) of an interval  $X$ .

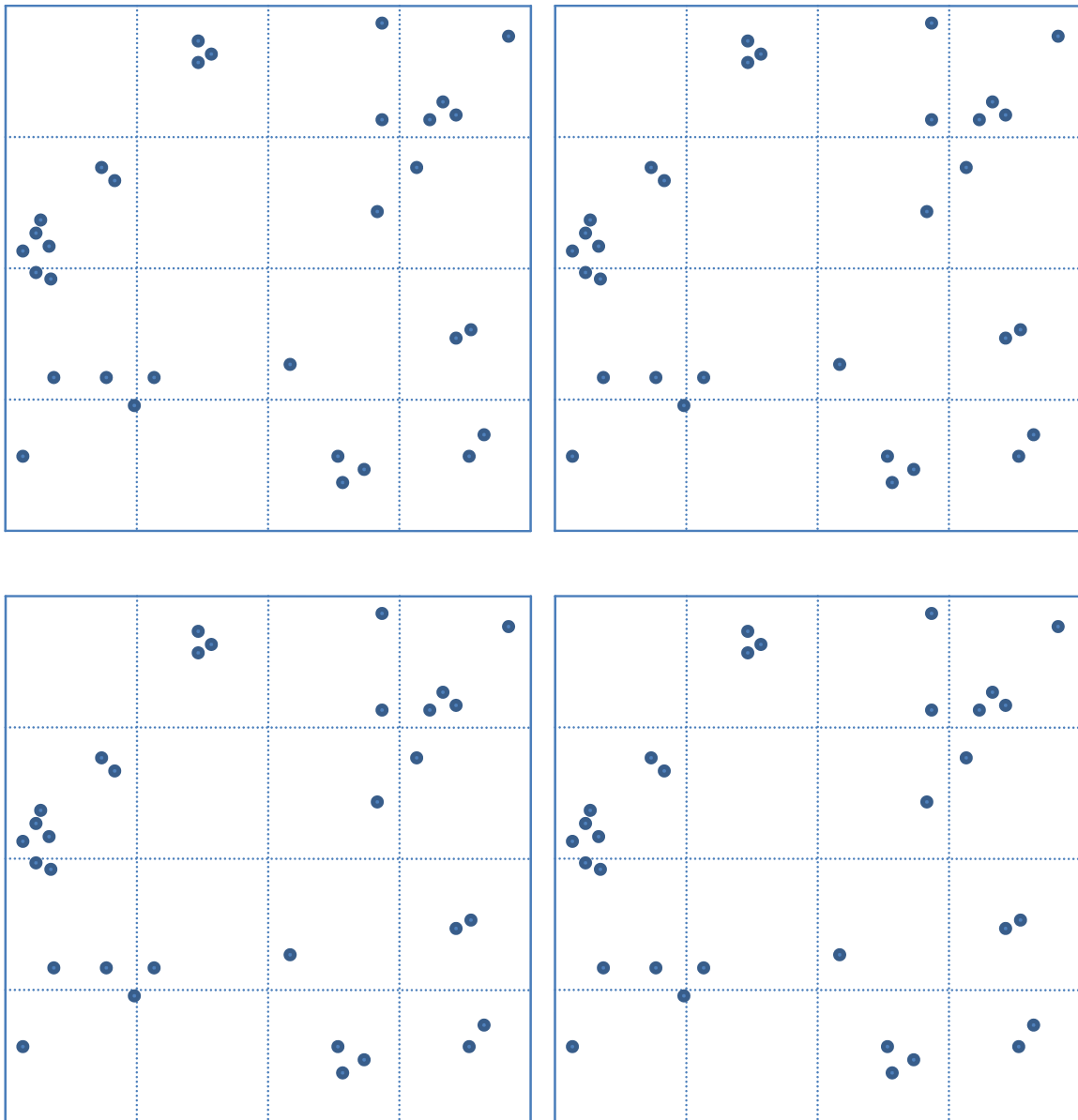
**Exercise 2-2**    *Spatial-Publish-Subscribe*



In the following the playing field is uniformly divided into a  $4 \times 4$  grid of square micro-zones. 32 objects are moving on this field with initial positions as shown above. The “Area of Interest” (AoI) of an object is circular with its radius corresponding to exactly half the side length of a grid cell. In the playing field shown in the figure above the AoIs of some objects are exemplary plotted.

- (a) Regard the object marked with *a*) in cell A4. In which micro-zones is this object first subscribed?
- (b) From which objects does *a*) initially get positional information? Which objects initially get positional information from *a*)?
- (c) To which micro-zones has *a*) to subscribe if it moves as shown in the figure?
- (d) To which micro-zones has the object marked with *b*) in cell B1 to subscribe (and un-subscribe) if it moves as shown in the figure?

**Exercise 2-3** *Spatial index structures*



In the following the dataset given above shall be indexed. Therefore use:

- (a) A quadtree with a bucket capacity of two objects. For the first splits subsidiary lines are given.
- (b) A kD-Tree with a capacity of four objects. Begin with a split of the x-axis.
- (c) An R-tree with a capacity of two objects. Build the R-tree with the sort-tile recursive algorithm.