Exercise 5-2  Probabilistic Balancing

Regarding another game players can choose between several different settings (e.g. races, classes, fractions) in the beginning. Let \( s_1, ..., s_n \) be such settings.

Assume that 1000 games were monitored each between a player with settings \( s_1 \) and a player with settings \( s_2 \) (briefly \( s_1 \) vs \( s_2 \)). 400 of those were won by the player with settings \( s_1 \). Is this game fair concerning the settings \( s_1 \) and \( s_2 \)? Therefore calculate the possibility of this observation assuming that the game was fair, i.e. that the winning chances for both players were always 50%.

Null hypothesis \( H_0 : p = 0.5 \)

If \( H_0 \) holds, the number of wins is binomial distributed with \( p = 0.5 \)

\[ \Rightarrow \#\text{wins} = B(1000, 0.5) \]

General: \( P(B(N, p) = i) = \binom{N}{i} \cdot p^i \cdot (1 - p)^{N-i} \)

Here: \( P(B(1000, 0.5) = i) = \binom{1000}{i} \cdot 0.5^{1000} \)

Probability, that \( \#\text{wins} \leq 400 \):

\[ P(B(1000, 0.5) \leq 400) = \sum_{i=0}^{400} P(B(1000, 0.5) = i) \]

Problem: Computation is exponential

Solution: Approximation of the binomial distribution with the gaussian distribution
**General overview**

Binomial distribution $B(n, p)$
- **Probability mass function**
- **Cumulative distribution function**

**Gaussian distribution** $N(\mu, \sigma^2)$
- **Probability density function**
- **Cumulative distribution function**

Expected value $\mu = n \times p$
Variance $\sigma^2 = n \times p \times (1 - p)$
Here

**Binomial**

\[ \text{wins} = B(1000, 0.5) \]

**Gaussian**

\[ \text{Expected value } \mu = n \times p \]
\[ \text{Variance } \sigma^2 = n \times p \times (1 - p) \]
\[ \text{wins} \approx N(500, 250) \]

\[ P(B(1000, 0.5) \leq 400) \approx P(N(500, 250) \leq 400) \]
Standardization of $N(500,250)$ to enable looking for its value in a quantile table, where values for $N(1,0)$ are listed.

\[
P(N(500,250) \leq 400) =
\]

\[
P(500 + N(0,250) \leq 400) =
\]

\[
P( N(0,250) \leq -100) =
\]

\[
P\left( N(0,1) \cdot \sqrt{250} \leq -100 \right) =
\]

\[
\frac{P(N(0,1) \leq \frac{-100}{\sqrt{250}})}{\sqrt{250}} =
\]

\[
P(N(0,1) \leq -6.32)
\]

Quantile table =>
\[
P(N(0,1) \leq -6.32) = 8.5 \times 10^{-8}
\]

Approximate probability for the observed results assuming a fair game.