Exercise 5-2 *Probabilistic Balancing*

Regarding another game players can choose between several different settings (e.g. races, classes, fractions) in the beginning. Let $s_1, \ldots, s_n$ be such settings.

Assume that 1000 games were monitored each between a player with settings s_1 and a player with settings s_2 (briefly s_1 vs s_2). 400 of those were won by the player with settings s_1. Is this game fair concerning the settings s_1 and s_2? Therefore calculate the possibility of this observation assuming that the game was fair, i.e. that the winning chances for both players were always 50%.

Null hypothesis $H_0: p = 0.5$

If H_0 holds, the number of wins is binomial distributed with $p = 0.5$

=> $\#wins = B(1000, 0.5)$

General: $P(B(N,p) = i) = \binom{N}{i} * p^i * (1-p)^{1000-i}$

Here: $P(B(1000,0.5) = i) = \binom{1000}{i} * 0.5^{1000}$

WANTED

Probability, that #wins ≤ 400 :

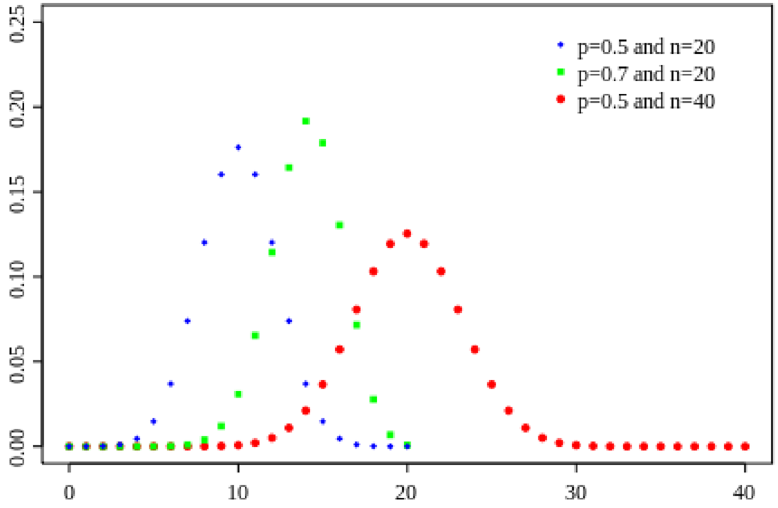
$$\mathrm{P}(B(1000,0.5) \leq 400) = \sum_{i=0}^{400} P(B(1000, 0.5) = i)$$

Problem: Computation is exponential

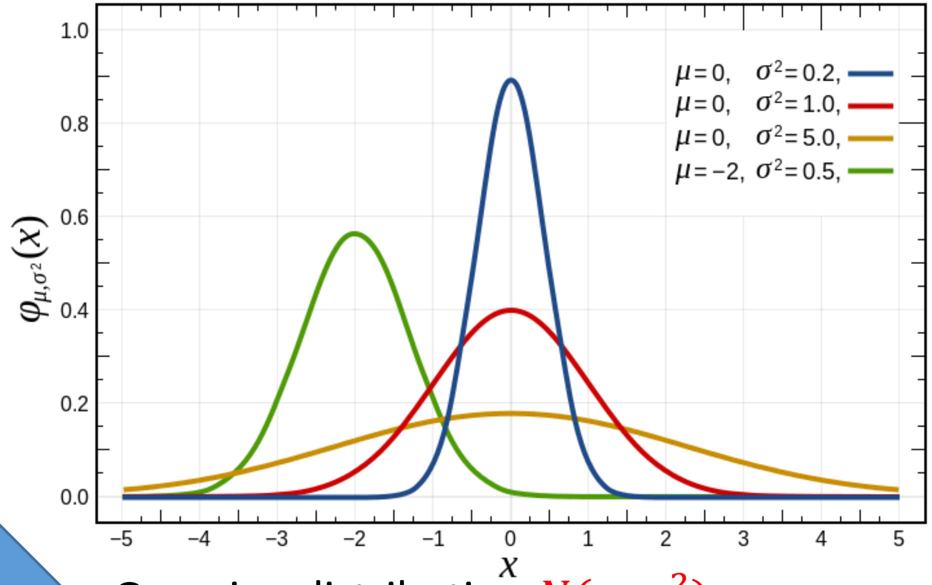
Solution: Approximation of the binomial distribution with the gaussian distribution

General overview

Binomial distribution $B(n,p)$
Probability mass function

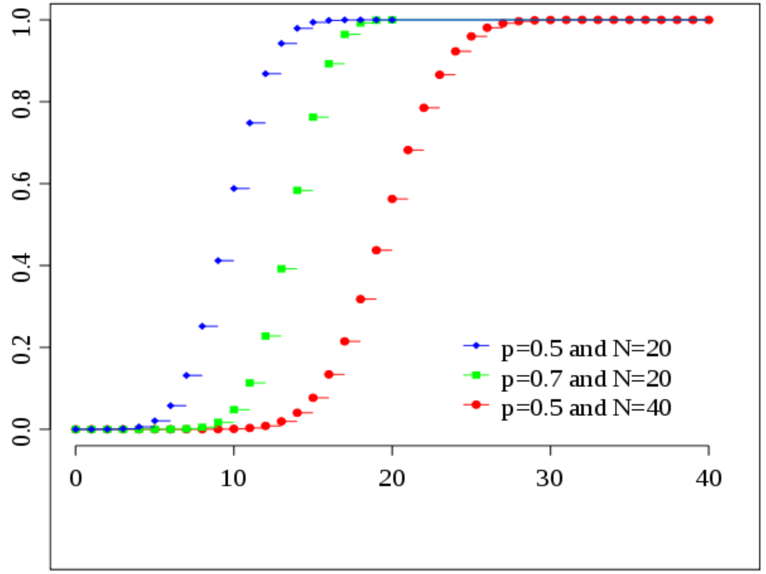


Gaussian distribution $N(\mu,\sigma^2)$
Probability density function

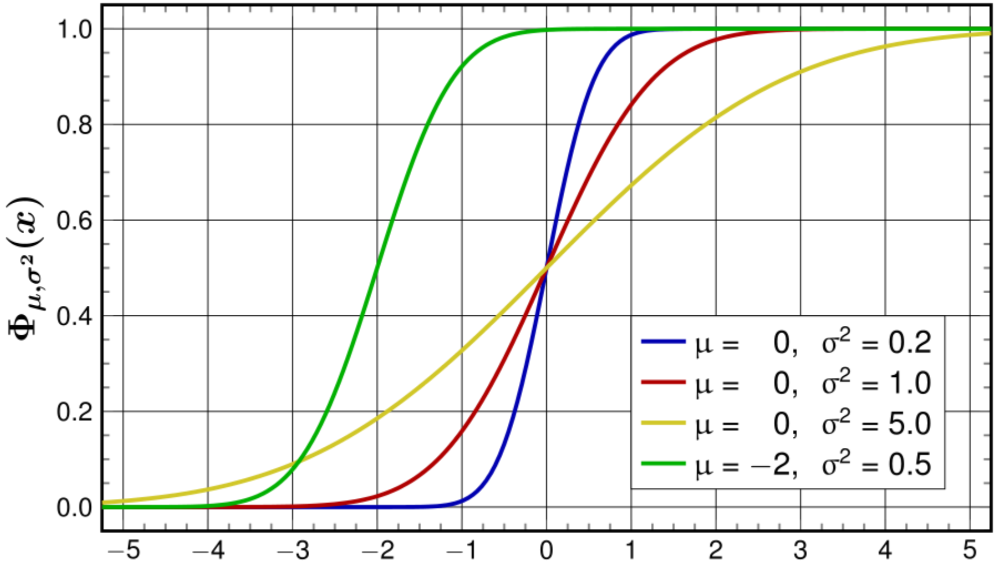


Expected value $\mu = n * p$
Variance $\sigma^2 = n * p * (1-p)$

Binomial distribution $B(n,p)$
Cumulative distribution function



Gaussian distribution $N(\mu,\sigma^2)$
Cumulative distribution function



Here

Binomial

$\#wins = B(1000, 0.5)$

Expected value $\mu = n * p$
Variance $\sigma^2 = n * p * (1 - p)$

Gaussian

$\#wins \approx N(500{,}250)$

$$\mathrm{P}(B(1000{,}0.5) \leq 400) \approx P(N(500{,}250) \leq 400)$$

Standardization of $N(500,250)$ to enable looking for its value in a quantile table, where values for $N(1,0)$ are listed.

$$P(N(500,250) \le 400) =$$

$$P(500 + N(0,250) \le 400) =$$

$$P(N(0,250) \le -100) =$$

$$P\left(N(0,1) * \sqrt{250} \le -100 \right) =$$

$$P\left(N(0,1) \le \frac{-100}{\sqrt{250}} \right) =$$

$$P(N(0,1) \le -6.32)$$

Quantile table =>

$$P(N(0,1) \le -6.32) = 8.5 * 10^{-8}$$

Approximate probability for the observed results assuming a fair game