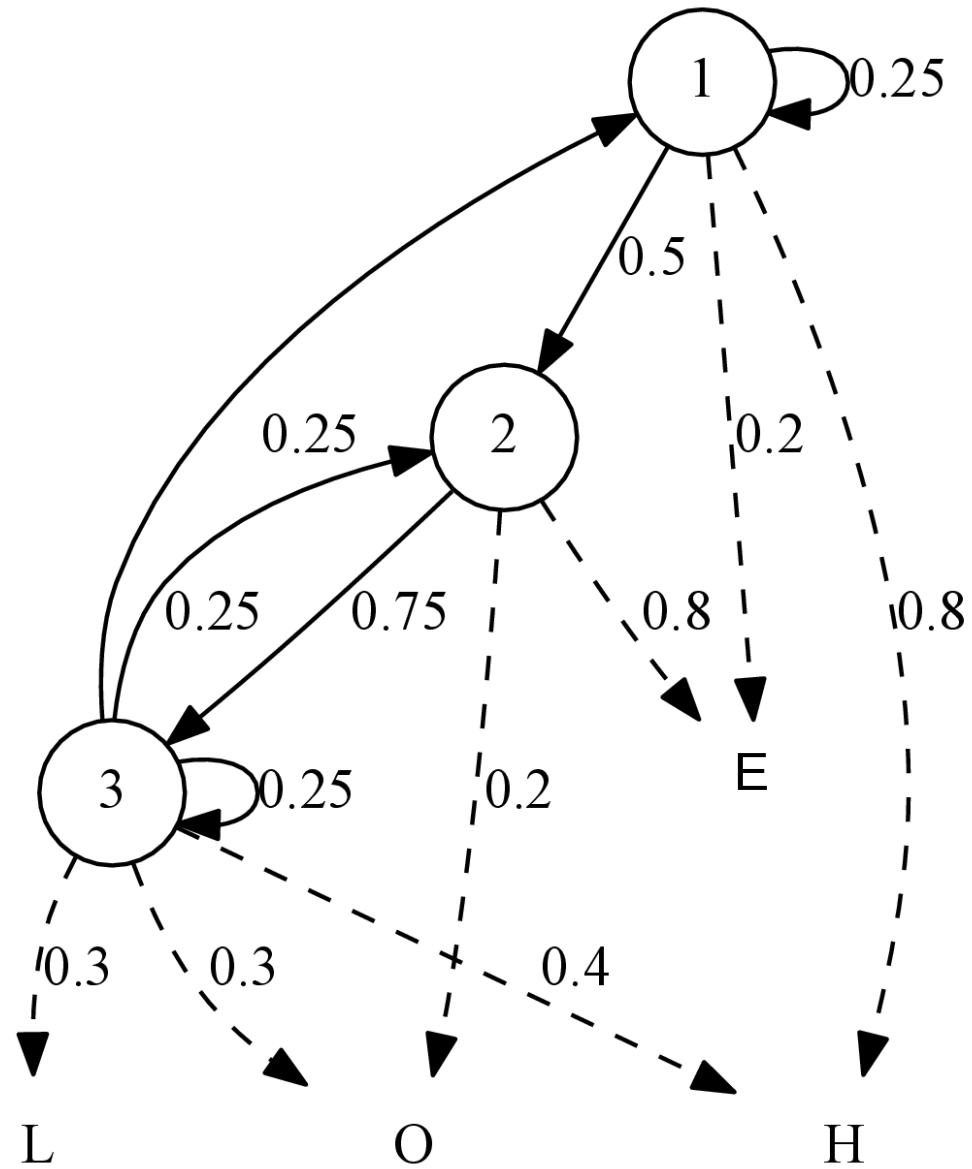
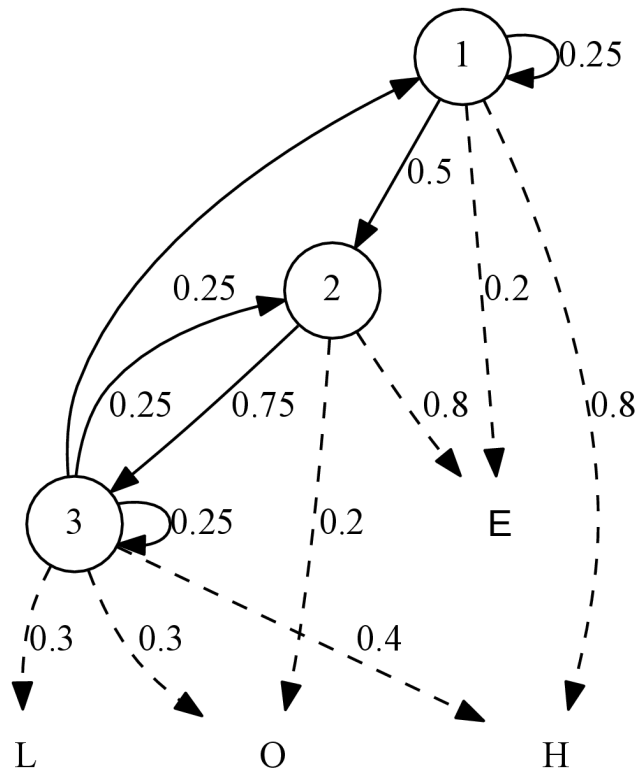


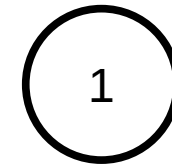
MMMO, 8-4



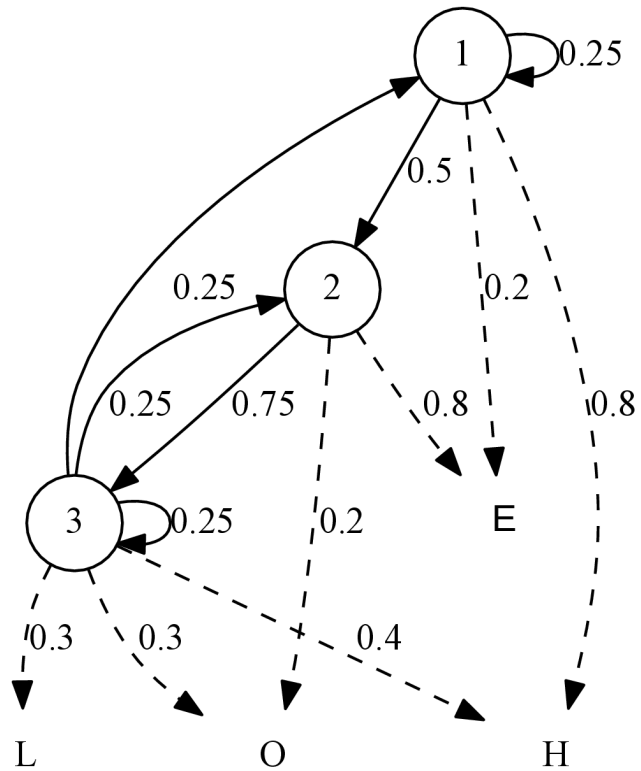


(a) Specify the set of states A and the set of observations B. Deduce the transition matrix D and the output matrix F from the model.

States:



Observation: „E”

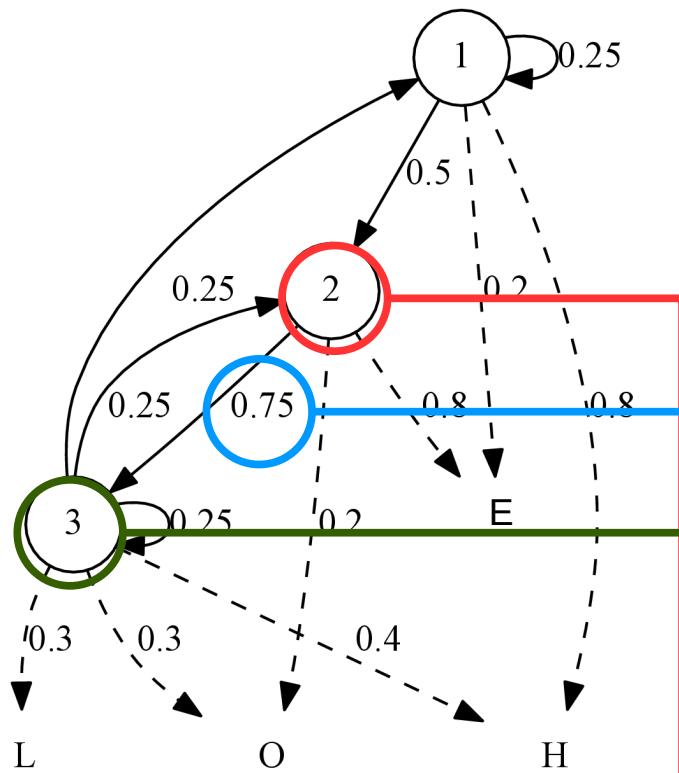


(a) Specify the set of states A and the set of observations B . Deduce the transition matrix D and the output matrix F from the model.

Set of states: $\{ 1, 2, 3 \}$

Set of observations: $\{ E, H, L, O \}$

(a) Specify the set of states A and the set of observations B. Deduce the transition matrix D and the output matrix F from the model.



Transition matrix

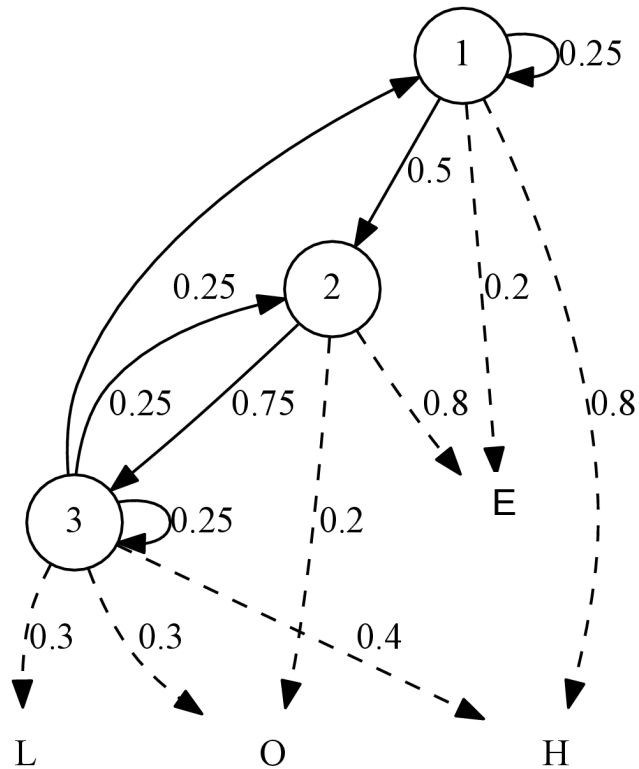
From:

To:

	1	2	3
1			
2		.75	
3			

Set of states: { 1, 2, 3 }

Set of observations: { E, H, L, O }



Set of states: $\{ 1, 2, 3 \}$

Set of observations: $\{ E, H, L, O \}$

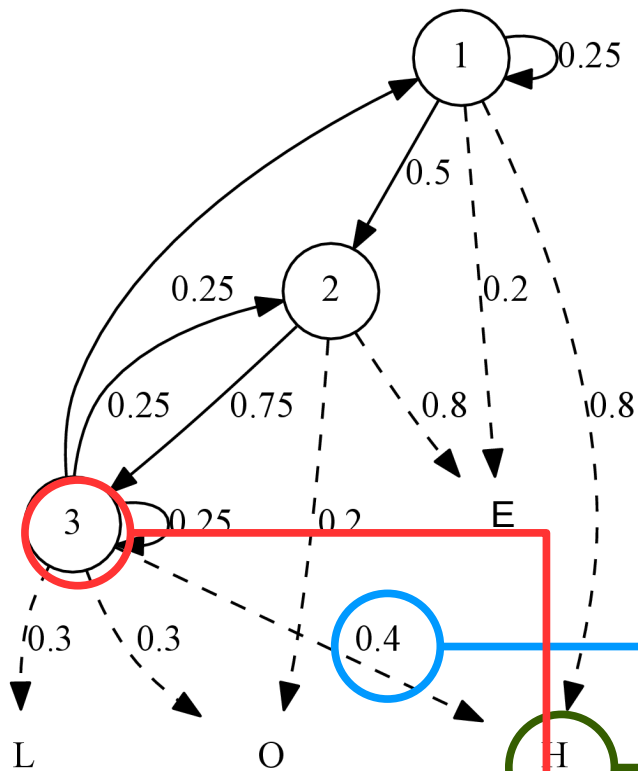
(a) Specify the set of states A and the set of observations B . Deduce the transition matrix D and the output matrix F from the model.

Transition matrix

		To:			
		—	1	2	3
From:	—	—	.33	.33	.33
	1	.25	.25	.5	—
	2	.25	—	—	.75
	3	.25	.25	.25	.25

Uniformly distributed

Row sum: 1



(a) Specify the set of states A and the set of observations B . Deduce the transition matrix D and the output matrix F from the model.

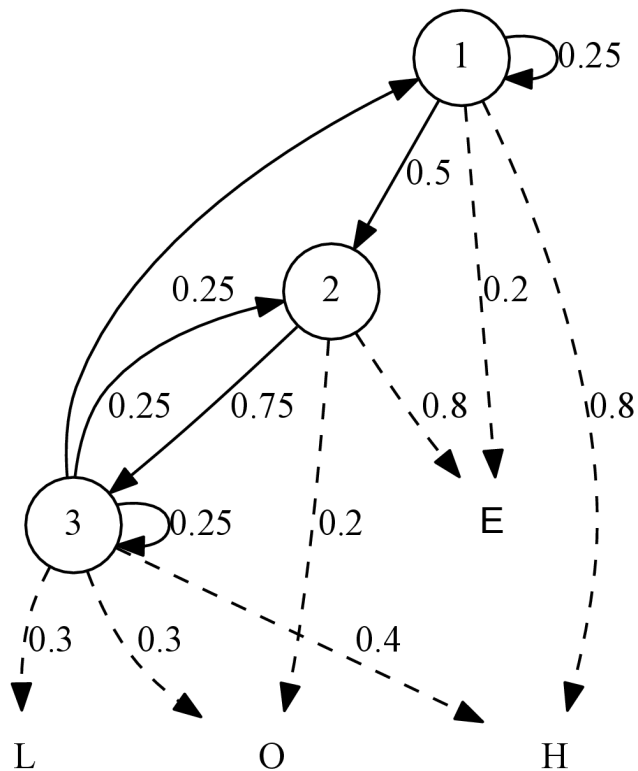
Output matrix:
From A to B

Set of states: $\{1, 2, 3\}$
Set of observations: $\{E, H, L, O\}$

Transition matrix

	—	1	2	3
—	—	.33	.33	.33
1	.25	.25	.5	—
2	.25	—	—	.75
3	.25	.25	.25	.25

	E	H	L	O
1				
2				
3				



(a) Specify the set of states A and the set of observations B . Deduce the transition matrix D and the output matrix F from the model.

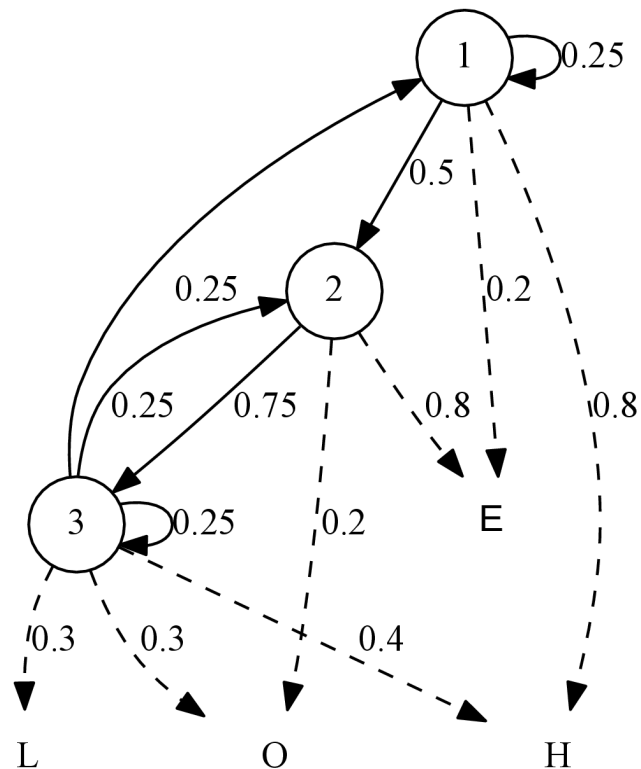
Output matrix:
From A to B

	E	H	L	O
1	.2	.8	—	—
2	.8	—	—	.2
3	—	.4	.3	.3

Set of states: $\{ 1, 2, 3 \}$
Set of observations: $\{ E, H, L, O \}$

Transition matrix

	—	1	2	3
—	—	.33	.33	.33
1	.25	.25	.5	—
2	.25	—	—	.75
3	.25	.25	.25	.25



(a) Specify the set of states A and the set of observations B . Deduce the transition matrix D and the output matrix F from the model.

Set of states: $\{1, 2, 3\}$

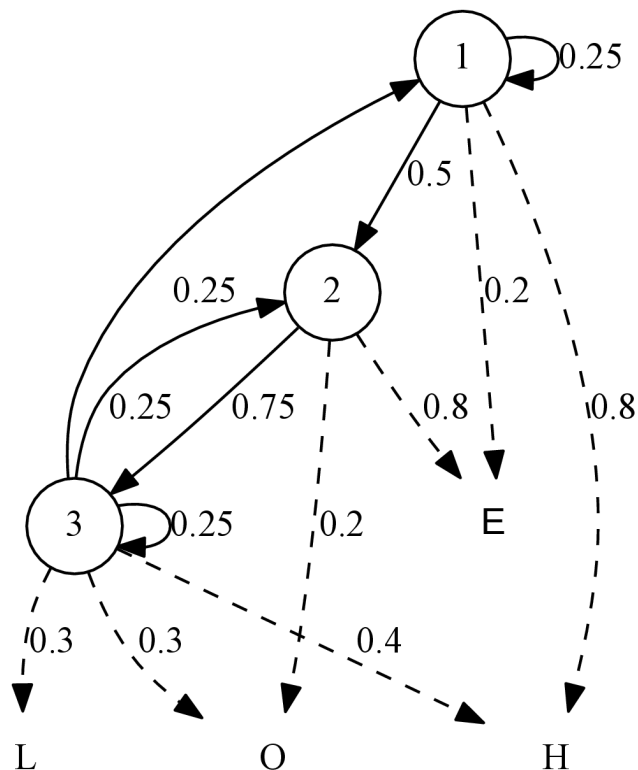
Set of observations: $\{E, H, L, O\}$

Transition Matrix

	—	1	2	3
—	—	.33	.33	.33
1	.25	.25	.5	—
2	.25	—	—	.75
3	.25	.25	.25	.25

Output matrix:

	E	H	L	O
1	.2	.8	—	—
2	.8	—	—	.2
3	—	.4	.3	.3



Transition matrix

	—	1	2	3
—	—	.33	.33	.33
1	.25	.25	.5	—
2	.25	—	—	.75
3	.25	.25	.25	.25

(b) Calculate the probability that the observation $O_1 = \{H, E, L, L, O\}$ is generated by the HMM.

Sequences of states which can produce the sequence $\{H, E, L, L, O\}$:

1-2-3-3-2
1-2-3-3-3
3-2-3-3-2
3-2-3-3-3

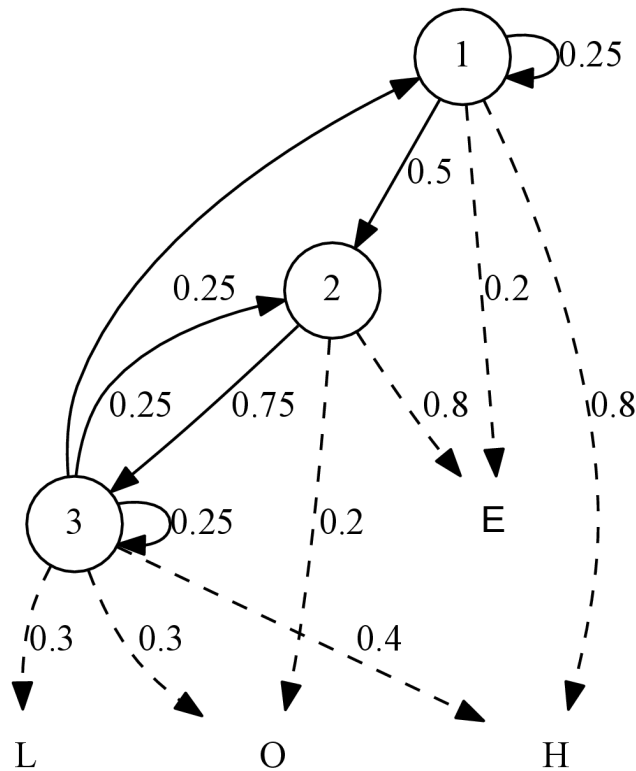
$$P(1|-) \times P(2|1) \times P(3|2) \times P(3|3) \times P(2|3) \times P(-|2) \\ = .33 \times .5 \times .75 \times .25 \times .25 \times .25 = .001933594$$

$$P(H|1) \times P(E|2) \times P(L|3) \times P(L|3) \times P(O|2) \\ = .8 \times .8 \times .3 \times .3 \times .2 = .01152$$

$$P(\text{HELLO}) = P(12332) \times P(\text{HELLO} | 12332) \\ + P(12333) \times P(\text{HELLO} | 12333) \\ + P(32332) \times P(\text{HELLO} | 32332) \\ + P(32333) \times P(\text{HELLO} | 32333)$$

$$P(H|1) \times P(E|2) \times P(L|3) \times P(L|3) \times P(O|3) \\ = .8 \times .8 \times .3 \times .3 \times .3 = .01728$$

$$P(1|-) \times P(2|1) \times P(3|2) \times P(3|3) \times P(3|3) \times P(-|3) \\ = .33 \times .5 \times .75 \times .25 \times .25 \times .25 = .001933594$$



Transition matrix

	—	1	2	3
—	—	.33	.33	.33
1	.25	.25	.5	—
2	.25	—	—	.75
3	.25	.25	.25	.25

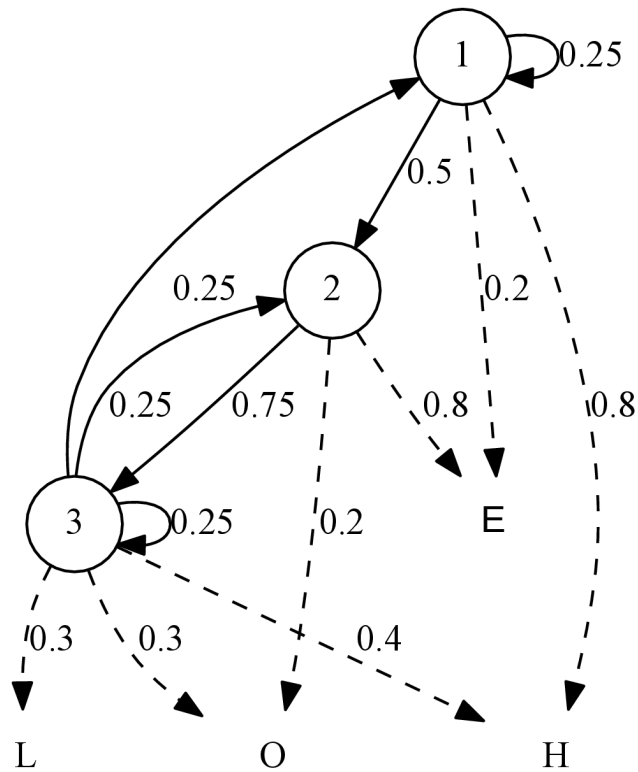
(b) Calculate the probability that the observation $O_1 = \{H, E, L, L, O\}$ is generated by the HMM.

Sequences of states which can produce the sequence $\{H, E, L, L, O\}$:

1-2-3-3-2
 1-2-3-3-3
 3-2-3-3-2
 3-2-3-3-3

$$\begin{aligned}
 P(\text{HELLO}) &= P(12332) \times P(\text{HELLO} \mid 12332) \\
 &\quad + P(12333) \times P(\text{HELLO} \mid 12333) \\
 &\quad + P(32332) \times P(\text{HELLO} \mid 32332) \\
 &\quad + P(32333) \times P(\text{HELLO} \mid 32333)
 \end{aligned}$$

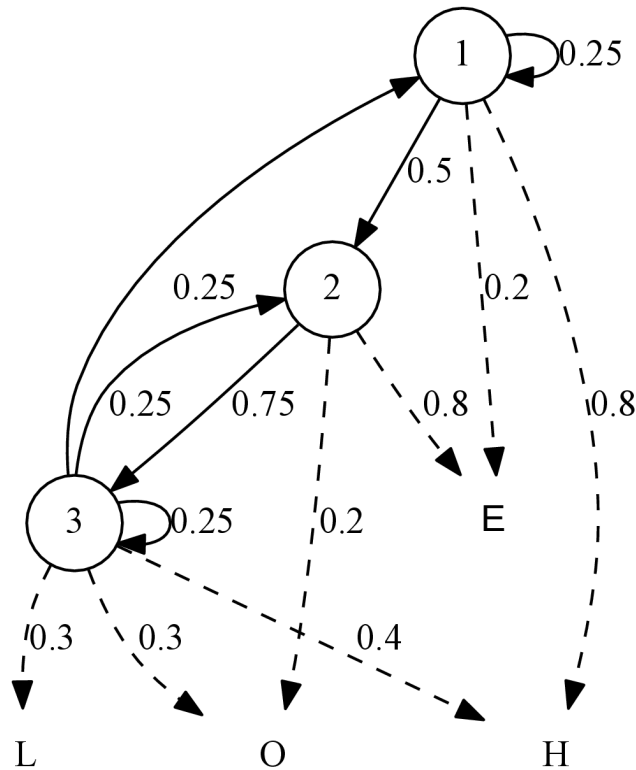
$$\begin{aligned}
 &= 0,001933594 \times 0,01152 \\
 &\quad + 0,001933594 \times 0,01728 \\
 &\quad + 0,000966797 \times 0,00576 \\
 &\quad + 0,000966797 \times 0,00864 \\
 &= 0,000069609
 \end{aligned}$$



(c) Which sequence $(s_1, s_2, \dots, s_k)$ with $s_i \in A$ explains the observation $O_1 = \{H, E, L, L, O\}$ best?

Sequences of states which can produce the sequence $\{H, E, L, L, O\}$:

1-2-3-3-2
 1-2-3-3-3
 3-2-3-3-2
 3-2-3-3-3



(c) Which sequence $(s_1, s_2, \dots, s_k)$ with $s_i \in A$ explains the observation $O_1 = \{H, E, L, L, O\}$ best?

Sequences of states which can produce the sequence $\{H, E, L, L, O\}$:

$$P(\text{HELLO} \wedge 1-2-3-3-2) = 0.000022275$$

$$\mathbf{P(\text{HELLO} \wedge 1-2-3-3-3) = 0.0000334125}$$

$$P(\text{HELLO} \wedge 3-2-3-3-2) = 0.00000556875$$

$$P(\text{HELLO} \wedge 3-2-3-3-3) = 0.00000835312$$

Efficient solution: **Viterbi-Algorithm**