Exercise 10-1  PCA
a) Please describe what a PCA aims for and under what circumstances it is most helpful.

b) Which possibly negative consequences might arise when applying PCA to a dataset of unknown structure?

Exercise 10-2  PCA
Consider the $X \in \mathbb{R}^{M\times N}$ matrix containing six data points $x_i \in \mathbb{R}^2$. Note that in contrast to the conventional representation, the patterns are held in columns here.

<table>
<thead>
<tr>
<th>dim 1</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>dim 2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

Conduct a PCA on the given data. Please state the eigenvectors, eigenvalues, covariance matrix and visualize the data before and after the PCA.

Exercise 10-3  Eigenfaces
The term eigenfaces describes the eigenvectors of a normalized covariance matrix.

a) Find the eigenfaces of the number dataset from the `numberMatrix.RTable`.

b) How many principal components are required to reconstruct the dataset? Are the eigenfaces sufficient for this purpose? Are all patterns reconstructable with equal quality?

Exercise 10-4  Soccer Ball PCA
The Caltech 101 dataset consists of more than 9000 images which have each been assigned to one out of 102 classes. We consider down-scaled images ($32 \times 32$-thumbnails) taken from the classes soccer ball and faces easy.

a) Conduct a PCA on the 64 soccer ball images. Can the images be reconstructed losslessly with only part of the principal components?

b) Now consider the faces easy dataset, consisting of 435 image. Can this dataset be adequately reconstructed using the principal components from part a)?

c) Now consider the dataset of part a) and the principal components of the dataset of part b). Does reconstructing the soccer balls from the faces’ components work?