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Machine Learning and Data Mining

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Solutions to Exercise Sheet 1

Exercise 1-3 Boolean Function as Perceptron

Consider the boolean function *or* ($\vee$) for two binary inputs.

- b) Given the above picture, guess weights for a perceptron (with outputs 0 and 1) such that the perceptron is a classifier for the $\vee$ function. Instead of using the $sign$ function, as in the lecture, use the Heaviside function f for classification:

$$f(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Possible Solution:

Intercept and gradient can easily be guessed from the illustration. For example, $1/2$ und -1 qualify as a solution, respectively.
The equation of the hyperplane is:

$$h = w_0 + w_1 x_1 + w_2 x_2$$

For $h = 0$, we obtain $0 = w_0 + w_1 x_1 + w_2 x_2$. Brought into the form of the equation of a straight line:

$$x_2 = -w_0/w_2 - (w_1/w_2)x_2.$$

If we set $w_2 = 1$, then $w_0 = -1/2$ and $w_1 = -1$ parametrize the above line. Damit ist $w = (-1/2, -1, 1)$.

- c) Initialize the weight vector as $w = (0, 0, 0)$ and learn the right weights employing the algorithm of the lecture and a learning rate $\eta = 0.2$. Use the following learning rule:

$$w_i \leftarrow w_i + \eta \cdot (y_j - \hat{y}_j) x_{i,j}$$

. Start training vector x_3 and proceed with increasing index (in contrast to the principle of random sampling).

Possible Solution:

sample (pattern)	y_i	weight update
x_3	$f(x_3^T w) = f(0) = 1$	–
x_0	$f(x_0^T w) = 1$	$w_0 \leftarrow 0 - 0.2 \cdot 1 = -0.2, w_1 = 0, w_2 = 0$
x_1	$f(x_1^T w) = 0$	$w_0 \leftarrow -0.2 + 0.2 \cdot 1 = 0, w_1 = 0, w_2 \leftarrow 0 + 0.2 \cdot 1 = 0.2$
x_2	$f(x_2^T w) = 1$	–
x_3	$f(x_3^T w) = 1$	–
x_0	$f(x_0^T w) = 1$	$w_0 \leftarrow 0 - 0.2 \cdot 1 = -0.2, w_1 = 0, w_2 = 0.2$
x_1	$f(x_1^T w) = 1$	–
x_2	$f(x_2^T w) = 0$	$w_0 \leftarrow -0.2 + 0.2 \cdot 1 = 0, w_1 = 0.2, w_2 = 0.2$
x_3	$f(x_3^T w) = 1$	–
x_0	$f(x_0^T w) = 1$	$w_0 \leftarrow 0 - 0.2 \cdot 1 = -0.2, w_1 = 0.2, w_2 = 0.2$
x_1	$f(x_1^T w) = 1$	–
x_2	$f(x_2^T w) = 1$	–
x_3	$f(x_3^T w) = 1$	–
x_0	$f(x_0^T w) = 0$	–