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Machine Learning and Data Mining

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Exercise Sheet 6

Presentation of Solutions to the Exercise Sheet on the 12.06.2014

Aufgabe 6-1 Determining the Optimal Separating Hyperplane

Determine the optimal separating hyperplane of the following dataset, partitioned into two classes A and B:

$$A = \left\{ p_1 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, p_2 = \begin{pmatrix} 3 \\ -1 \end{pmatrix}, p_3 = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}, p_4 = \begin{pmatrix} 2.5 \\ 3 \end{pmatrix}, p_5 = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \right\},$$
$$B = \left\{ p_6 = \begin{pmatrix} 0.5 \\ 1.5 \end{pmatrix}, p_7 = \begin{pmatrix} -1 \\ 3 \end{pmatrix}, p_4 = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix} \right\}$$

Instances of class A shall be labeled with 1, instances of class B with -1.

Visualize the result and name the support vectors. How wide is the margin?

Aufgabe 6-2 Lagrangian Multipliers

Consider the optimization problem (given $x \in \mathbb{R}^n$) of the form $f_i : \mathbb{R}^n \to \mathbb{R}$ and $g_j : \mathbb{R}^n \to \mathbb{R}$:

$$\begin{array}{lll} \min_{x\in\mathbb{R}^n} & f_0(x) & \\ \text{subject to} & f_i(x) \leq 0 & \text{für } i = 1, \ldots, m \\ & g_j(x) = 0 & \text{für } j = 1, \ldots, p \end{array}$$

f_0 is the objective function, f_i and g_j are the inequality and equality constraints, respectively. A Langrangian incorporates the constraints into the objective function and optimizes within the given limits:

$$\mathcal{L}(x, \gamma, \lambda) = f_0(x) + \textstyle\sum_{i=1}^{m} \gamma_i f_i(x) + \sum_{j=1}^{p} \lambda_j g_j(x) \,,$$

b.w.

where γ and λ are real vectors and *Lagrangian Multipliers*. They are also referred to as dual variables. All γ_i are to be ≥ 0, whereas the λ_j may be chosen freely.

Optimize the following problems for $n = 2$, employing a Lagrangian for $x_1, x_2 \in \mathbb{R}$, where $x_1 + x_2 = 20$,

a) maximizing $x_1 \cdot x_2$.

b) maximizing $x_1^2 + x_2^2$.

c) maximizing $e^{-(5x_1 - x_2)^2}$.

Find the corresponding Lagrangian and minimize it w.r.t. the objectives x_1, and x_2. Then, the original condition may be applied.

Aufgabe 6-3 Minimal Surface

A closed cardboard box shall have the capacity of 36 cm^3. Additionally, the width of its base shall have triple the length of its base.

Compute length, width and height of the box with the smallest surface.