

In CF vectors, the correct formula is

$$R^2 = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - x_0)^2} \Rightarrow \sqrt{\frac{SS}{N} - \left(\frac{LS}{N}\right)^2}$$

LS not SS
(as in the slides)

Proof

$$R^2 = \underbrace{\frac{1}{N} \sum_{i=1}^N x_i^2}_A - \underbrace{\frac{2}{N} \sum_{i=1}^N x_i x_0}_B + \underbrace{\frac{1}{N} \sum_{i=1}^N x_0^2}_C$$

$$A: \frac{1}{N} \left(\sum_{i=1}^N x_i^2 \right) = \frac{1}{N} \cdot SS = \frac{SS}{N}$$

$$B: \frac{2}{N} \cdot \sum_{i=1}^N x_i x_0 = \frac{2}{N} \sum_{i=1}^N x_i \cdot \left(\frac{\sum_{i=1}^N x_i}{N} \right) = \frac{2}{N^2} \sum_{i=1}^N x_i \left(\sum_{i=1}^N x_i \right)$$

$$= \frac{2}{N^2} \cdot \sum_{i=1}^N x_i \cdot \sum_{i=1}^N x_i = \frac{2 \left(\sum_{i=1}^N x_i \right)^2}{N^2} = 2 \left(\frac{\sum_{i=1}^N x_i}{N} \right)^2$$

$$C: \frac{1}{N} \sum_{i=1}^N x_0^2 = \frac{1}{N} \sum_{i=1}^N \left(\frac{\sum_{i=1}^N x_i}{N} \right)^2 = \frac{1}{N^3} \sum_{i=1}^N \left(\sum_{i=1}^N x_i \right)^2 =$$

$$= \frac{N}{N^3} \cdot \left(\sum_{i=1}^N x_i \right)^2 = \frac{\left(\sum_{i=1}^N x_i \right)^2}{N^2} = \left(\frac{\sum_{i=1}^N x_i}{N} \right)^2$$

$$A - B + C = \frac{SS}{N} - 2 \left(\frac{\sum_{i=1}^N x_i}{N} \right)^2 + \left(\frac{\sum_{i=1}^N x_i}{N} \right)^2$$

$$= \frac{SS}{N} - \left(\frac{\sum_{i=1}^N x_i}{N} \right)^2 = \frac{SS}{N} - \left(\frac{LS}{N} \right)^2$$