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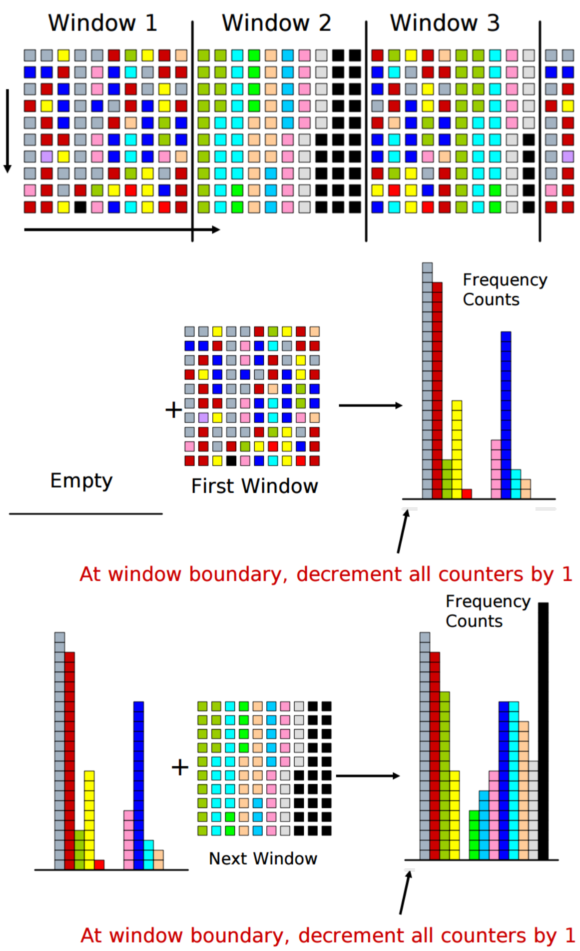
Knowledge Discovery in Databases II
SS 2018

Exercise 6: Data Stream Clustering

Exercise 6-1 Lossy Counting

Before stream clustering, let's take a look at a more fundamental task in stream: count the occurrence of objects in a stream and output objects with a count larger or equal to some given threshold: $minSup \times L$, where L is the length of the stream up to now and $minSup$ is the given threshold (minimum support).

Lossy Counting is one of the basic algorithms that solve this problem. Given the windows size as $w = \frac{1}{\epsilon}$, the lossy counting algorithm works as the follows: cut stream into windows, process one window a time and prune histogram entries with 0 counts at each window boundary. The illustration is given below:



Please prove that

(a) the maximum count error (the maximum difference between the real count and the estimated count) of the lossy counting algorithm is ϵL

Let W be the current windows id, then $\frac{1}{\epsilon}W = L \Rightarrow W = \epsilon L$. The count error of a given object is happened due to the reduction of count at each window boundary. Thus, including the current windows boundary, the maximum count error is $\Delta = W = \epsilon L$.

(b) the memory consumption, i.e., the number of entries stored in the histogram, is $O(\frac{1}{\epsilon}\log(\epsilon L))$. (Optional)

Let W be the current window id. For each $i \in [1, W]$, let d_i denote the number of entries in the histogram H which corresponds to window $W - i + 1$.

Thus, the item in the stream corresponding to such entry must occur at least i times in window $B - i + 1$ through W; otherwise, it would have been deleted. Since the size of each window is w, we have:

$$\sum_{i=1}^{j} i d_i \leq jw \quad for\ j = 1, 2, \ldots, W \tag{1}$$

Now we want to prove:

$$\sum_{i=1}^{j} d_i \leq \sum_{i=1}^{j} \frac{w}{i} \tag{2}$$

By induction:

- For $j = 1$, this is true.
- Assume it is true for $j = 1, 2, \ldots, p-1$, then for $j = p$, adding the inequality (1) for $j = p$ to the aggregation of all $p - 1$ instances of the inequality (2) gives us:

  $$\sum_{i=1}^{p} i d_i + \sum_{i=1}^{1} d_i + \sum_{i=1}^{2} d_i + \cdots + \sum_{i=1}^{p-1} d_i \leq pw + \sum_{i=1}^{1} \frac{w}{i} + \sum_{i=1}^{2} \frac{w}{i} + \cdots + \sum_{i=1}^{p-1} \frac{w}{i}$$

  The left hand side can be written as:

  $$1 \cdot d_1 + 2 \cdot d_2 + \cdots + p \cdot d_p + (d_1) + (d_1 + d_2) + \cdots + (d_1 + d_2 + \cdots + d_{p-1}) = p \cdot d_1 + p \cdot d_2 + \cdots + p \cdot d_p = p \sum_{i=1}^{p} d_i$$

  Similarly, the right hand side can be written as:

  $$\frac{1 \cdot w}{1} + \frac{2 \cdot w}{2} + \cdots + \frac{p \cdot w}{p} + (\frac{w}{1}) + (\frac{w}{1} + \frac{w}{2}) + \cdots + (\frac{w}{1} + \frac{w}{2} + \cdots + \frac{w}{p-1}) = \frac{pw}{1} + \frac{pw}{2} + \cdots + \frac{pw}{p} = p \sum_{i=1}^{p} \frac{w}{i}$$

  Then we have:

  $$p \sum_{i=1}^{p} d_i \leq p \sum_{i=1}^{p} \frac{w}{i}$$
  $$\Rightarrow \sum_{i=1}^{p} d_i \leq \sum_{i=1}^{p} \frac{w}{i}$$

Thus the memory consumption at window W is $|H| = \sum_{i=1}^{W} d_i \leq \sum_{i=1}^{W} \frac{w}{i} = \frac{1}{\epsilon} \log W = \frac{1}{\epsilon} \log \epsilon L$

(Here the inequality $\sum_{i=1}^{W} \frac{1}{i} \leq \log W$ is used, as it is the harmonic series.)