Exercise 3-1 Principal Component Analysis

Consider the following example on principal axis transformation.

Given:

\[ X = \{ (-3, -2), (-2, -1), (-1, 0), (0, 1), (1, 2), (2, 3), \\
    (-2, -2), (-1, -1), (0, 0), (1, 1), (2, 2), \\
    (-2, -3), (-1, -2), (0, -1), (1, 0), (2, 1), (3, 2) \} \]

(a) Calculate the covariance matrix \( M \).
(b) Calculate eigenvalues and eigenvectors of \( M \).
(c) Determine the smallest eigenvalue and remove its corresponding eigenvector. The remaining eigenvector is the basis of a new sub-space.
(d) Transform all vectors in \( X \) in this new sub-space by expressing all vectors in \( X \) in this new basis.

Exercise 3-2 Principal Component Analysis

Conduct a principal axis transformation on the following data set:

- \( A(1, 0, 3) \), \( B(0, 0, 3) \), \( C(1, 0, 1) \), \( D(0, 0, 1) \)

What problem comes up? How can it be solved?

Exercise 3-3 Singular Value Decomposition

Another approach to feature reduction is Singular Value Decomposition. Given a Matrix \( M \) and its SVD decomposition:

\[ M = T \ast S \ast D' \]

with

\[ M = \begin{bmatrix} 1 & 2 \\ 6 & 3 \\ 0 & 2 \end{bmatrix} \quad T = \begin{bmatrix} -0.2707 & 0.5458 \\ -0.9509 & -0.2797 \\ -0.1497 & 0.7899 \end{bmatrix} \]
\[ S = \begin{bmatrix} 7.0257 & 0 \\ 0 & 2.1539 \end{bmatrix} \quad D = \begin{bmatrix} -0.8507 & -0.5257 \\ -0.5257 & 0.8507 \end{bmatrix} \]

Reduce to one dimension using the approach described in the lecture script.