

**Knowledge Discovery in Databases II**  
SS 2017

**Exercise 3: Dimensionality Reduction**

**Exercise 3-1 Principal Component Analysis**

Consider the following example on principal axis transformation.

Given:

$$X = \{(-3, -2), (-2, -1), (-1, 0), (0, 1), (1, 2), (2, 3), \\ (-2, -2), (-1, -1), (0, 0), (1, 1), (2, 2), \\ (-2, -3), (-1, -2), (0, -1), (1, 0), (2, 1), (3, 2)\}$$

- Calculate the covariance matrix  $M$ .
- Calculate eigenvalues and eigenvectors of  $M$ .
- Determine the smallest eigenvalue and remove its corresponding eigenvector. The remaining eigenvector is the basis of a new sub-space.
- Transform all vectors in  $X$  in this new sub-space by expressing all vectors in  $X$  in this new basis.

**Exercise 3-2 Principal Component Analysis**

Conduct a principal axis transformation on the following data set:

- $A(1, 0, 3), B(0, 0, 3), C(1, 0, 1), D(0, 0, 1)$

What problem comes up? How can it be solved?

**Exercise 3-3 Singular Value Decomposition**

Another approach to feature reduction is Singular Value Decomposition. Given a Matrix  $M$  and its SVD decomposition:

$$M = T * S * D'$$

with

$$M = \begin{bmatrix} 1 & 2 \\ 6 & 3 \\ 0 & 2 \end{bmatrix} \quad T = \begin{bmatrix} -0.2707 & 0.5458 \\ -0.9509 & -0.2797 \\ -0.1497 & 0.7899 \end{bmatrix} \\ S = \begin{bmatrix} 7.0257 & 0 \\ 0 & 2.1539 \end{bmatrix} \quad D = \begin{bmatrix} -0.8507 & -0.5257 \\ -0.5257 & 0.8507 \end{bmatrix}$$

Reduce to one dimension using the approach described in the lecture script.