Ludwig Maximilians Universität München
Institut für Informatik
Lehr- und Forschungseinheit für Datenbanksysteme

Skript zur Vorlesung

Knowledge Discovery in Databases

im Wintersemester 2009/2010

Kapitel 6: Outlier Detection

basiert auf Tutorial von Hans-Peter Kriegel, Peer Kröger, Arthur Zimek:
Outlier Detection Techniques (PAKDD-09, Bangkok, Thailand)

http://www.dbs.ifi.lmu.de/Lehre/KDD

6 Outlier Detection

Übersicht

Was ist ein Outlier?

Definition nach Hawkins [Hawkins 1980]:

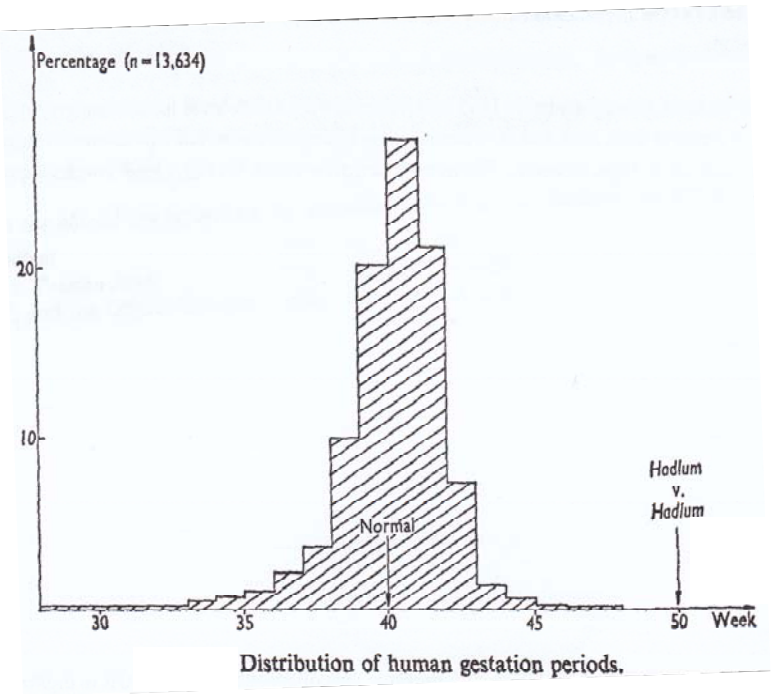
"Ein Outlier ist eine *Beobachtung*, die sich von den anderen *Beobachtungen* so deutlich unterscheidet, daß man denken könnte, sie sei von einem anderen Mechanismus generiert worden."

Was meint "Mechanismus"?

- Intuition aus der Statistik: "erzeugender Mechanismus" ist ein (statistischer) Prozess.
- Abnormale Daten (outlier) zeigen eine verdächtig geringe Wahrscheinlichkeit, aus diesem Prozess zu stammen.

Beispiel: Hadlum vs. Hadlum (1949) [Barnett 1978]

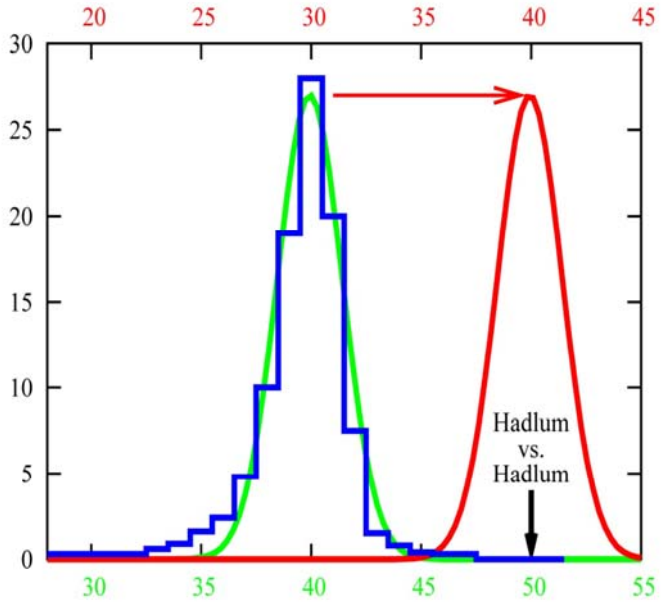
- Geburt eines Kindes von Mrs. Hadlum 349 Tage nachdem Mr. Hadlum zum Militärdienst abwesend war.
- Durchschnittliche Dauer einer menschlichen Schwangerschaft ist 280 Tage (40 Wochen)
- Ist eine Schwangerschaftsdauer von 349 Tagen ein Outlier?



Distribution of human gestation periods.

Beispiel: Hadlum vs. Hadlum (1949) [Barnett 1978]

- Blau: statistische Beobachtungsbasis (13634 erhobene Schwangerschaften)
- Grün: angenommener zugrundeliegender Gauss-Prozess
  - sehr geringe Wahrscheinlichkeit, dass die Geburt aus diesem Prozess stammt
- Rot: Annahme von Mr. Hadlum (ein anderer Gauss-Prozess, in dem die Schwangerschaft später beginnt, ist für die Geburt verantwortlich)
  - unter dieser Annahme hat die Schwangerschaftsdauer einen Durchschnittswert und höchst-mögliche Wahrscheinlichkeit



Anwendungsgebiete:

- Betrugsentdeckung
  - Kaufverhalten mit einer Kreditkarte ändert sich, wenn die Karte gestohlen wurde
  - Ungewöhnliche Kauf-Muster können Kreditkarten-Mißbrauch anzeigen
- Medizin
  - Ungewöhnliche Symptome oder Test-Ergebnisse können mögliche gesundheitliche Probleme eines Patienten anzeigen
  - Ob ein bestimmtes Testergebnis ungewöhnlich ist, kann von anderen Eigenschaften des Patienten abhängen (z.B. Geschlecht, Alter, Gewicht, ...)
- Öffentliches Gesundheitswesen
  - Auftauchen einer bestimmten Krankheit (z.B. Tetanus) verstreut über verschiedene Krankenhäuser einer Stadt zeigt Probleme mit dem zugehörigen Impfprogramm an
  - Ob das Auftreten der Krankheit unnormal ist hängt von verschiedenen Aspekten ab, z.B. Häufigkeit, räumliche Korrelation etc.

Anwendungsgebiete:

- Sport Statistiken
  - In vielen Sportarten werden diverse Parameter aufgezeichnet, um die Leistung eines Spielers zu bewerten
  - Außergewöhnliche (in positivem wie negativem Sinne) Spieler können durch ungewöhnliche Werte bestimmt werden
  - Manchmal ist nur eine Teilmenge der Parameter ungewöhnlich
- Entdecken von Messfehlern
  - Daten aus Sensoren (z.B. in einem wissenschaftlichen Experiment) können Meßfehler enthalten
  - Ungewöhnliche Werte können ein Hinweis auf Meßfehler sein
  - Solche Meßfehler aus den Daten zu entfernen, kann wichtig sein für erfolgreiche Datenanalyse und Data Mining

„One person's noise could be another person's signal."

Diskussion der Intuition von Hawkins

- Daten sind gewöhnlich multivariat (mehr-dimensional)
  => Basis-Modell ist univariat (ein-dimensional)
- Ein Datensatz stammt oft aus mehr als einem erzeugenden Prozess
  => Basis-Model nimmt nur einen einzelnen genuinen erzeugenden Mechanismus an
- Anomalien können eine andere Klasse von Objekten sein (aus einem anderen Prozess erzeugt), die nicht besonders selten sind
  => Basis-Model nimmt an, dass Outlier sehr selten sind

Eine große Zahl von Methoden wurde entwickelt, um über die Basis-Annahmen hinauszugelangen. Dabei liegen jedoch stets andere, oft nicht explizite Annahmen zugrunde.

Generelle Szenarien der Anwendung:

- supervised
  - in manchen Anwendungsgebieten gibt es Trainingsdaten mit normalen und ungewöhnlichen Fällen
  - es kann mehrere normale und ungewöhnliche Klassen geben
  - meist ist das Klassifikationsproblem unbalanziert
- semi-supervised
  - in manchen Szenarien gibt es Trainingsdaten nur für die normale oder nur für die ungewöhnliche Klasse
- unsupervised
  - in den meisten Szenarien gibt es keine Trainingsdaten

In dieser Vorlesung konzentrieren wir uns auf das unsupervised Szenario.

Erkennung von Outliern

- Nebenprodukt von Clustering?
- Manche Cluster-Algorithmen ordnen nicht jeden Punkt einem Cluster zu, sondern lassen "Noise" übrig.
- Idee: Wende Cluster-Verfahren an, betrachte Noise als Outlier.

- Problem:
  - Clustering Algorithmen sind daraufhin entwickelt und optimiert, Cluster zu finden.
  - Qualität der Outlier Detection hängt von Qualität der Cluster-Struktur und der Eignung des Clustering Algorithmus für diese Struktur ab.
  - Mehrere Outlier, die einander ähnlich sind, bilden eventuell auch selbst ein (kleines) Cluster, können also nicht entdeckt werden.

Klassifikation von Outlier Detection Algorithmen

- Globaler vs. lokaler Ansatz:
  Wird die "Outlierness" bestimmt bezüglich des gesamten Datensatzes (global) oder nur bezüglich einer Auswahl?
- Labeling vs. Scoring
  Bestimmt der Algorithmus den Outlier-Grad eines Punktes (Scoring) oder wird für jeden Punkt eine Entscheidung getroffen (Label: Outlier/kein Outlier)
- Eigenschaften des Outlier Modells
  Auf welchen Eigenschaften beruht die Modellierung von "Outlierness"

- Global vs. Lokal
  - bezieht sich auf die Auflösung der Referenzmenge bezüglich derer die "Outlierness" bestimmt wird
  - Globale Ansätze:
    - Referenzmenge enthält gesamten Datensatz
    - Basis-Annahme: nur ein einziger (normaler) erzeugender Mechanismus
    - Grundlegendes Problem: Outlier sind auch in Referenzmenge und verfälschen die Ergebnisse
  - Lokale Ansätze:
    - Referenzmenge enthält nur eine (kleine) Teilmenge des Datensatzes
    - keine Annahme über Anzahl der Mechanismen
    - Grundlegendes Problem: wie ist eine geeignete Referenzmenge zu bestimmen?
  - Beachte: Manche Ansätze liegen dazwischen
    - Auflösung der Referenzmenge wird im Verfahren variiert

- Labeling vs. Scoring
  - bezieht sich auf das Ergebnis, das der Algorithmus liefert
  - Labeling Ansätze:
    - binäre Entscheidung
    - Daten-Objekt wird als Outlier markiert oder als normal
  - Scoring Ansätze:
    - kontinuierlicher Output: für jedes Objekt wird ein Score geliefert (z.B. die Wahrscheinlichkeit, ein Outlier zu sein)
    - Objekte können nach ihrem Score geordnet werden
  - Beachte:
    - Viele Scoring-Ansätze bestimmen nur die top-n Outlier (Parameter n wird durch Benutzer angegeben)
    - Scoring-Ansätze können grundsätzlich in Labeling-Ansätze transformiert werden, wenn ein geeigneter Grenzwert angegeben werden kann, dessen Überschreitung zum Label "Outlier" führt

- Klassen von zugrundeliegenden Modellen
  - Statistisches Modell
    - Überlegung:
      - Wende ein Modell an, das die normalen Daten statistisch beschreibt (z.B. Gauss-Verteilung)
      - Outlier sind Punkte, die nicht gut zu diesem Modell passen (eine geringe Erzeugungswahrscheinlichkeit haben)
    - Beispiele:
      - Wahrscheinlichkeitstests basierend auf statistischen Modellen
      - Tiefen-basierte Ansätze
      - Deviation-based Ansätze
      - Manche Subspace Outlier Detection Ansätze

- Modellierung durch räumliche Nähe
  - Überlegung:
    - Untersuche die räumliche Nachbarschaft jedes Punktes im Datenraum
    - Wenn die Nachbarschaft deutlich andere Struktur (z.B. geringere Dichte) aufweist als die Nachbarschaften von anderen Punkten, kann der betreffende Punkt als Outlier angesehen werden.
  - Beispiele:
    - Distanz-basierte Ansätze
    - Dichte-basierte Ansätze
    - Manche Subspace Outlier Detection Ansätze

- Modellierung durch Winkel-Spektrum
  - Überlegung:
    - Bestimme das Spektrum paarweiser Winkel zwischen einem gegebenen Punkt und anderen (alle? Auswahl?) Punkten
    - Outlier sind Punkte, die eine geringe Varianz haben

Im Folgenden:

Orientierung an den verschiedenen Modellierungen

6 Outlier Detection

Übersicht

Statistisches Modell

Modellierung durch räumliche Nähe

Anpassung verschiedener Modelle an spezielles Problem

6.2 Statistical Tests

General idea

- Given a certain kind of statistical distribution (e.g., Gaussian)
- Compute the parameters assuming all data points have been generated by such a statistical distribution (e.g., mean and standard deviation)
- Outliers are points that have a low probability to be generated by the overall distribution (e.g., deviate more than 3 times the standard deviation from the mean)

Basic assumption

- Normal data objects follow a (known) distribution and occur in a high probability region of this model
- Outliers deviate strongly from this distribution

6.2 Statistical Tests

A huge number of different tests are available differing in

- Type of data distribution (e.g. Gaussian)
- Number of variables, i.e., dimensions of the data objects (univariate/multivariate)
- Number of distributions (mixture models)
- Parametric versus non-parametric (e.g. histogram-based)

Example on the following slides

- Gaussian distribution
- Multivariate
- 1 model
- Parametric

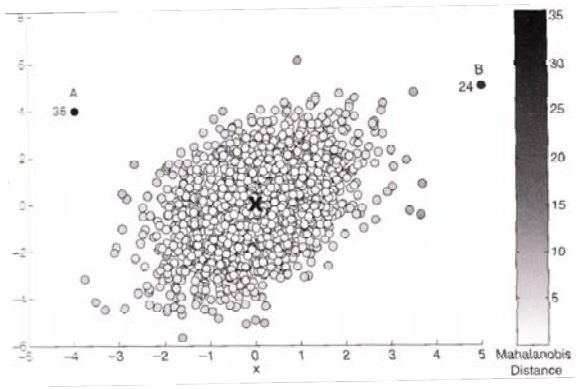
6.2 Statistical Tests

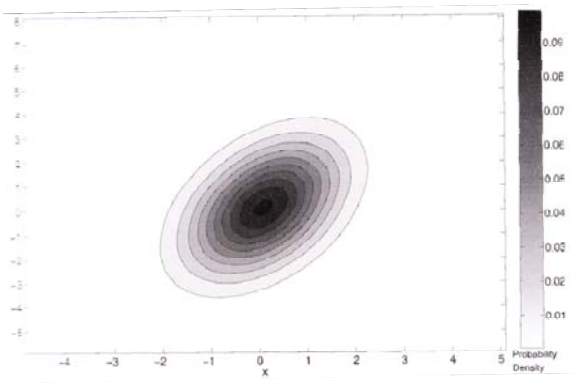
Probability density function of a multivariate normal distribution

$$N(x) = \frac{1}{\sqrt{(2\pi)^d \, |\Sigma|}} e^{-\frac{(x-\mu)^T \Sigma^{-1} (x-\mu)}{2}}$$

- μ is the mean value of all points (usually data are normalized such that $\mu=0$)
- Σ is the covariance matrix from the mean
- $MDist(x,\mu) = (x-\mu)^T \Sigma^{-1} (x-\mu)$ is the Mahalanobis distance of point x to μ
- MDist follows a χ^2-distribution with d degrees of freedom (d = data dimensionality)
- All points x, with $MDist(x,\mu) > \chi^2(0{,}975)$ [$\approx 3 \cdot \sigma$]

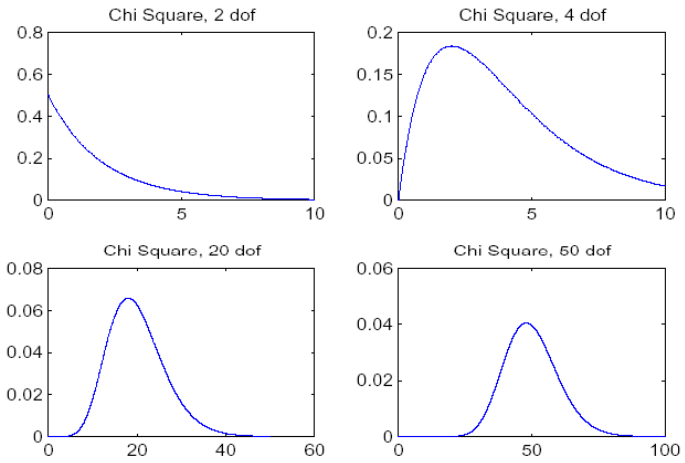
Visualization (2D) [Tan et al. 2006]





Problems

- Curse of dimensionality
  - The larger the degree of freedom, the more similar the MDist values for all points



x-axis: observed *MDist* values
y-axis: frequency of observation

6.2 Statistical Tests

Problems (cont.)

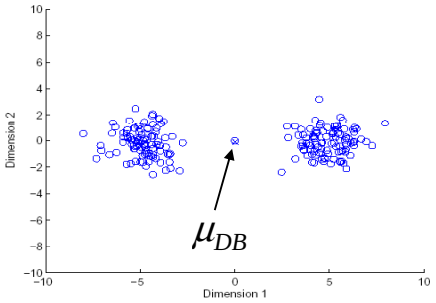
- Robustness
  - Mean and standard deviation are very sensitive to outliers
  - These values are computed for the complete data set (including potential outliers)
  - The *MDist* is used to determine outliers although the *MDist* values are influenced by these outliers

  => Minimum Covariance Determinant [Rousseeuw and Leroy 1987]

  minimizes the influence of outliers on the Mahalanobis distance

Discussion

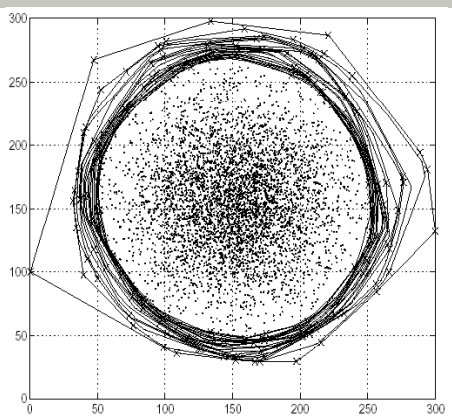
- Data distribution is fixed
- Low flexibility (no mixture model)
- Global method
- Outputs a label but can also output a score



6.3 Depth-based Approaches

General idea

- Search for outliers at the border of the data space but independent of statistical distributions
- Organize data objects in convex hull layers
- Outliers are objects on outer layers



Picture taken from [Johnson et al. 1998]

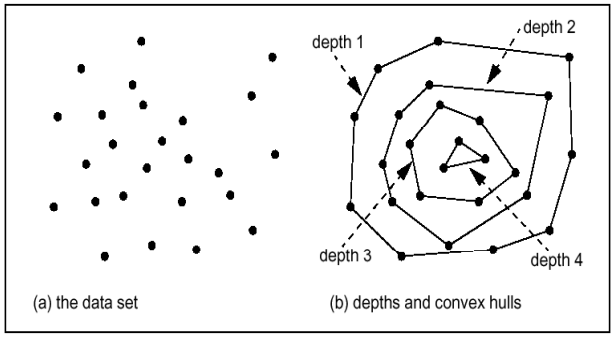
Basic assumption

- Outliers are located at the border of the data space
- Normal objects are in the center of the data space

6.3 Depth-based Approaches

Model [Tukey 1977]

- Points on the convex hull of the full data space have depth = 1
- Points on the convex hull of the data set after removing all points with depth = 1 have depth = 2
- …
- Points having a depth ≤ *k* are reported as outliers



Picture taken from [Preparata and Shamos 1988]

6.3 Depth-based Approaches

Sample algorithms

- ISODEPTH [Ruts and Rousseeuw 1996]
- FDC [Johnson et al. 1998]

Discussion

- Similar idea like classical statistical approaches (k = 1 distributions) but independent from the chosen kind of distribution
- Convex hull computation is usually only efficient in 2D / 3D spaces
- Originally outputs a label but can be extended for scoring easily (take depth as scoring value)
- Uses a global reference set for outlier detection

6.4 Deviation-based Approaches

General idea

- Given a set of data points (local group or global set)
- Outliers are points that do not fit to the general characteristics of that set, i.e., the variance of the set is minimized when removing the outliers

Basic assumption

- Outliers are the outermost points of the data set

6.4 Deviation-based Approaches

Model [Arning et al. 1996]

- Given a smoothing factor SF(I) that computes for each $I \subseteq DB$ how much the variance of DB is decreased when I is removed from DB
- The outliers are the elements of the **exception set** $E \subseteq DB$ for which the following holds:

$$SF(E) \geq SF(I) \quad \text{for all } I \subseteq DB$$

Discussion:

- Similar idea like classical statistical approaches (k = 1 distributions) but independent from the chosen kind of distribution
- Naïve solution is in $O(2^n)$ for n data objects
- Heuristics like random sampling or best first search are applied
- Applicable to any data type (depends on the definition of SF)
- Originally designed as a global method
- Outputs a labeling

6 Outlier Detection

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Statistisches Modell

Modellierung durch räumliche Nähe

Anpassung verschiedener Modelle an spezielles Problem

6.5 Distance-based Approaches

General Idea

- Judge a point based on the distance(s) to its neighbors
- Several variants proposed

Basic Assumption

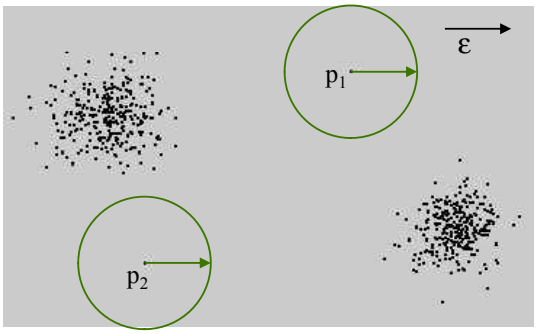
- Normal data objects have a dense neighborhood
- Outliers are far apart from their neighbors, i.e., have a less dense neighborhood

DB(ε,π)-Outliers

- Basic model [Knorr and Ng 1997]
  - Given a radius ε and a percentage π
  - A point *p* is considered an outlier if less than π percent of all other points have a distance to *p* greater than ε

$$OutlierSet(\varepsilon,\pi) = \{p \mid \frac{Card(\{q \in DB \mid dist(p,q) > \varepsilon\})}{Card(DB)} < \pi\}$$

range-query with radius ε



- Algorithms
  - Index-based [Knorr and Ng 1998]
    - Compute distance range join using spatial index structure
    - Exclude point from further consideration if its ε-neighborhood contains more than $Card(DB) \cdot \pi$ points
  - Nested-loop based [Knorr and Ng 1998]
    - Divide buffer in two parts
    - Use second part to scan/compare all points with the points from the first part
  - Grid-based [Knorr and Ng 1998]
    - Build grid such that any two points from the same grid cell have a distance of at most ε to each other
    - Points need only compared with points from neighboring cells

6.5 Distance-based Approaches

- Deriving intensional knowledge [Knorr and Ng 1999]
  - Relies on the DB(ε,π)-outlier model
  - Find the minimal subset(s) of attributes that explains the "outlierness" of a point, i.e., in which the point is still an outlier
  - Example
    - Identified outliers

Player Name	Power-play Goals	Short-handed Goals	Game-winning Goals	Game-tying Goals	Games Played
MARIO LEMIEUX	31	8	8	0	70
JAROMIR JAGR	20	1	12	1	82
JOHN LECLAIR	19	0	10	2	82
ROD BRIND'AMOUR	4	4	5	4	82

    - Derived intensional knowledge (sketch)

```
MARIO LEMIEUX:
  (i)  An outlier in the 1-D space of Power-play goals
  (ii) An outlier in the 2-D space of Short-handed goals and
       Game-winning goals
       (No player is exceptional on Short-handed goals alone;
        No player is exceptional on Game-winning goals alone.)
ROD BRIND'AMOUR:
  (i)  An outlier in the 1-D space of Game-tying goals
JAROMIR JAGR:
  (i)  An outlier in the 2-D space of Short-handed goals and
       Game-winning goals
       (No player is exceptional on Short-handed goals alone;
        No player is exceptional on Game-winning goals alone.)
  (ii) An outlier in the 2-D space of Power-play goals and
       Game-winning goals
```


6.5 Distance-based Approaches

Outlier scoring based on *k*NN distances

- General models
  - Take the *k*NN distance of a point as its outlier score [Ramaswamy et al 2000]
  - Aggregate the distances of a point to all its 1NN, 2NN, …, *k*NN as an outlier score [Angiulli and Pizzuti 2002]
- Algorithms
  - General approaches
    - Nested-Loop
      - » Naïve approach:
        For each object: compute *k*NNs with a sequential scan
      - » Enhancement: use index structures for *k*NN queries
    - Partition-based
      - » Partition data into micro clusters
      - » Aggregate information for each partition (e.g. minimum bounding rectangles)
      - » Allows to prune micro clusters that cannot qualify when searching for the *k*NNs of a particular point

6.5 Distance-based Approaches

- Sample Algorithms (computing top-*n* outliers)
  - Nested-Loop [Ramaswamy et al 2000]
    - Simple NL algorithm with index support for *k*NN queries
    - Partition-based algorithm (based on a clustering algorithm that has linear time complexity)
    - Algorithm for the simple *k*NN-distance model
  - Linearization [Angiulli and Pizzuti 2002]
    - Linearization of a multi-dimensional data set using space-fill curves
    - 1D representation is partitioned into micro clusters
    - Algorithm for the average *k*NN-distance model
  - ORCA [Bay and Schwabacher 2003]
    - NL algorithm with randomization and simple pruning
    - Pruning: if a point has a score greater than the top-*n* outlier so far (cut-off), remove this point from further consideration
      => non-outliers are pruned
      => works good on randomized data (can be done in linear time)
      => worst-case: naïve NL algorithm
    - Algorithm for both *k*NN-distance models and the DB(ε,π)-outlier model

6.5 Distance-based Approaches

- Sample Algorithms (cont.)
  - RBRP [Ghoting et al. 2006],
    - Idea: try to increase the cut-off as quick as possible => increase the pruning power
    - Compute approximate *k*NNs for each point to get a better cut-off
    - For approximate *k*NN search, the data points are partitioned into micro clusters and *k*NNs are only searched within each micro cluster
    - Algorithm for both *k*NN-distance models
  - Further approaches
    - Also apply partitioning-based algorithms using micro clusters [McCallum et al 2000], [Tao et al. 2006]
    - Approximate solution based on reference points [Pei et al. 2006]
- Discussion
  - Output can be a scoring (*k*NN-distance models) or a labeling (*k*NN-distance models and the DB(ε,π)-outlier model)
  - Approaches are local (resolution can be adjusted by the user via ε or *k*)

Variant

- Outlier Detection using In-degree Number [Hautamaki et al. 2004]
  - Idea
    - Construct the *k*NN graph for a data set
      - » Vertices: data points
      - » Edge: if $q \in k\text{NN}(p)$ then there is a directed edge from p to q
    - A vertex that has an indegree less or equal T (user defined threshold) is an outlier
  - Discussion
    - The indegree of a vertex in the *k*NN graph equals to the number of reverse kNNs (R*k*NN) of the corresponding point
    - The R*k*NNs of a point p are those data objects having p among their *k*NNs
    - Intuition of the model: outliers are
      - » points that are among the *k*NNs of less than T other points
      - » have less than T R*k*NNs
    - Outputs an outlier label
    - Is a local approach (depending on user defined parameter *k*)

Resolution-based outlier factor (ROF) [Fan et al. 2006]

- Model
  - Depending on the resolution of applied distance thresholds, points are outliers or within a cluster
  - With the maximal resolution *Rmax* (minimal distance threshold) all points are outliers
  - With the minimal resolution *Rmin* (maximal distance threshold) all points are within a cluster
  - Change resolution from *Rmax* to *Rmin* in certain steps: points change from being outlier to being a member of a cluster
  - Cluster is defined similar as in DBSCAN [Ester et al 1996] as a transitive closure of *r*-neighborhoods (where *r* is the current resolution)
  - ROF value

$$ROF(p) = \sum_{R\min \le r \le R\max} \frac{clusterSize_{r-1}(p) - 1}{clusterSize_r(p)}$$

- Discussion
  - Outputs a score (the ROF value)
  - Resolution is varied automatically from local to global

General idea

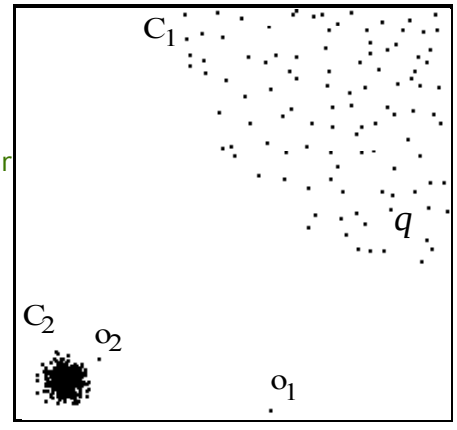
- Compare the density around a point with the density around its local neighbors
- The relative density of a point compared to its neighbors is computed as an outlier score
- Approaches also differ in how to estimate density

Basic assumption

- The density around a normal data object is similar to the density around its neighbors
- The density around an outlier is considerably different to the density around its neighbors

Local Outlier Factor (LOF) [Breunig et al. 1999], [Breunig et al. 2000]

- Motivation:
  - Distance-based outlier detection models have problems with different densities
  - How to compare the neighborhood of points from areas of different densities?
  - Example
    - DB(ε,π)-outlier model
      - » Parameters ε and π cannot be chosen so that o_2 is an outlier but none of the points in cluster C_1 (e.g. q) is an outlier
    - Outliers based on kNN-distance
      - » kNN-distances of objects in C_1 (e.g. q) are larger than the kNN-distance of o_2

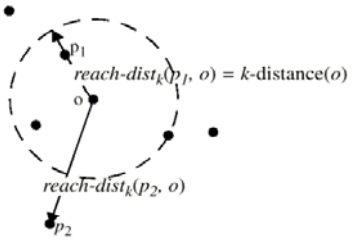


- Solution: consider relative density

6.6 Density-based Approaches

- Model
  - Reachability distance
    - Introduces a smoothing factor

$$reach\text{-}dist_k(p,o) = \max\{k\text{-distance}(o), dist(p,o)\}$$



  - Local reachability distance (lrd) of point *p*
    - Inverse of the average reach-dists of the *k*NNs of *p*

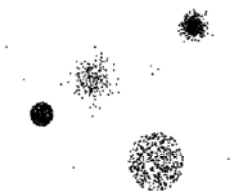
$$lrd_k(p) = 1 / \left(\frac{\sum_{o \in kNN(p)} reach\text{-}dist_k(p,o)}{Card(kNN(p))} \right)$$

  - Local outlier factor (LOF) of point *p*
    - Average ratio of lrds of neighbors of *p* and lrd of *p*

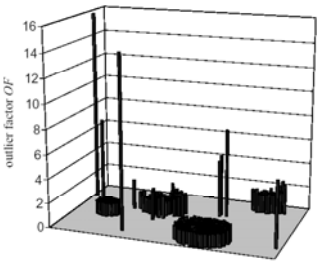
$$LOF_k(p) = \frac{\sum_{o \in kNN(p)} \frac{lrd_k(o)}{lrd_k(p)}}{Card(kNN(p))}$$

6.6 Density-based Approaches

- Properties
  - LOF ≈ 1: point is in a cluster (region with homogeneous density around the point and its neighbors)
  - LOF >> 1: point is an outlier



Data set



LOFs (*MinPts* = 40)

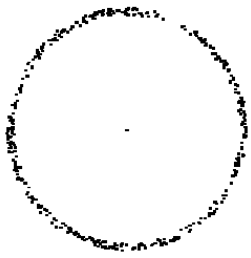
- Discussion
  - Choice of *k* (*MinPts* in the original paper) specifies the reference set
  - Originally implements a local approach (resolution depends on the user's choice for *k*)
  - Outputs a scoring (assigns an LOF value to each point)

Variants of LOF

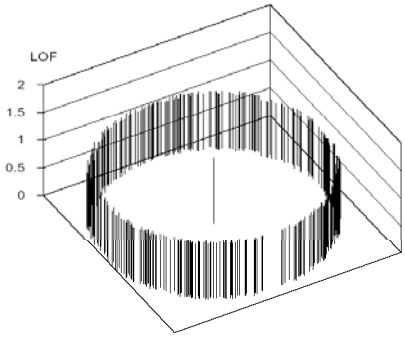
- Mining top-*n* local outliers [Jin et al. 2001]
  - Idea:
    - Usually, a user is only interested in the top-*n* outliers
    - Do not compute the LOF for all data objects => save runtime
  - Method
    - Compress data points into micro clusters using the CFs of BIRCH [Zhang et al. 1996]
    - Derive upper and lower bounds of the reachability distances, lrd-values, and LOF-values for points within a micro clusters
    - Compute upper and lower bounds of LOF values for micro clusters and sort results w.r.t. ascending lower bound
    - Prune micro clusters that cannot accommodate points among the top-*n* outliers (*n* highest LOF values)
    - Iteratively refine remaining micro clusters and prune points accordingly

Variants of LOF (cont.)

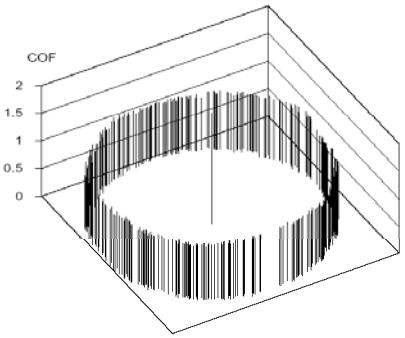
- Connectivity-based outlier factor (COF) [Tang et al. 2002]
  - Motivation
    - In regions of low density, it may be hard to detect outliers
    - Choose a low value for *k* is often not appropriate
  - Solution
    - Treat "low density" and "isolation" differently
  - Example



Data set



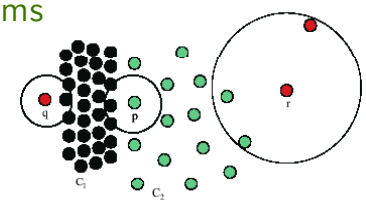
LOF



COF

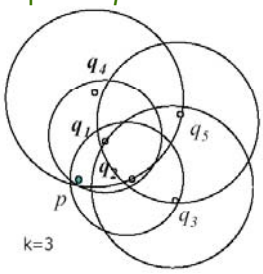
Influenced Outlierness (INFLO) [Jin et al. 2006]

- Motivation
  - If clusters of different densities are not clearly separated, LOF will have problems



Point *p* will have a higher LOF than points *q* or *r* which is counter intuitive

- Idea
  - Take symmetric neighborhood relationship into account
  - Influence space (*k*IS(*p*)) of a point *p* includes its kNNs (kNN(*p*)) and its reverse kNNs (RkNN(*p*))



kIS(*p*) = kNN(p) ∪ RkNN(*p*))

= {q_1, q_2, q_4}

- Model
  - Density is simply measured by the inverse of the *k*NN distance, i.e.,

    den(*p*) = 1/*k*-distance(*p*)

  - Influenced outlierness of a point p

$$INFLO_k(p) = \frac{\sum_{o \in kIS(p)} den(o) \,/\, Card(kIS(p))}{den(p)}$$

  - INFLO takes the ratio of the average density of objects in the neighborhood of a point *p* (i.e., in *k*NN(*p*) ∪ R*k*NN(*p*)) to *p*'s density

- Proposed algorithms for mining top-*n* outliers
  - Index-based
  - Two-way approach
  - Micro cluster based approach

- Properties
  - Similar to LOF
  - INFLO ≈ 1: point is in a cluster
  - INFLO >> 1: point is an outlier
- Discussion
  - Outputs an outlier score
  - Originally proposed as a local approach (resolution of the reference set kIS can be adjusted by the user setting parameter k)

Local outlier correlation integral (LOCI) [Papadimitriou et al. 2003]

- Idea is similar to LOF and variants
- Differences to LOF
  - Take the ε-neighborhood instead of *k*NNs as reference set
  - Test multiple resolutions (here called "granularities") of the reference set to get rid of any input parameter
- Model
  - ε-neighborhood of a point p: $N(p,\varepsilon) = \{q \mid dist(p,q) \leq \varepsilon\}$
  - Local density of an object p: number of objects in $N(p,\varepsilon)$
  - Average density of the neighborhood

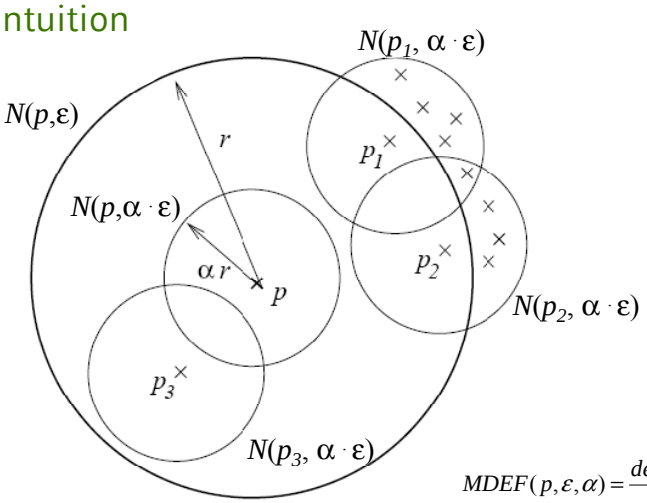
$$den(p,\varepsilon,\alpha) = \frac{\sum_{q \in N(p,\varepsilon)} Card(N(q,\alpha \cdot \varepsilon))}{Card(N(p,\varepsilon))}$$

  - Multi-granularity Deviation Factor (MDEF)

$$MDEF(p,\varepsilon,\alpha) = \frac{den(p,\varepsilon,\alpha) - Card(N(p,\alpha \cdot \varepsilon))}{den(p,\alpha,\varepsilon)} = 1 - \frac{Card(N(p,\alpha \cdot \varepsilon))}{den(p,\alpha,\varepsilon)}$$

6.6 Density-based Approaches

- Intuition



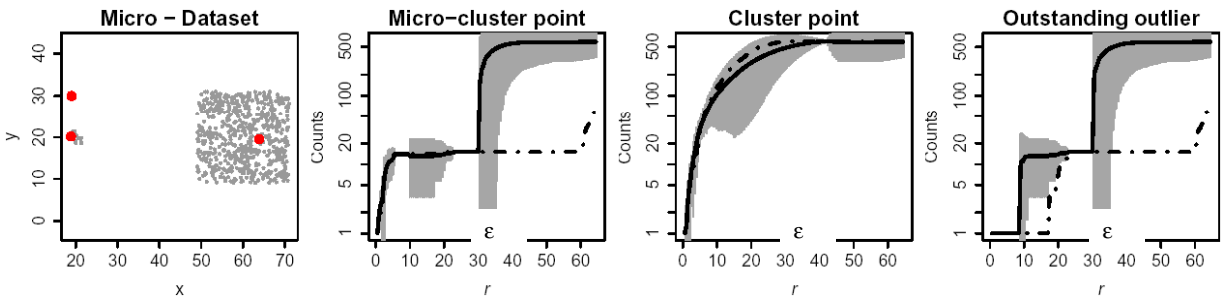
$$den(p,\varepsilon,\alpha) = \frac{\sum_{q \in N(p,\varepsilon)} Card(N(q,\alpha \cdot \varepsilon))}{Card(N(p,\varepsilon))}$$

$$MDEF(p,\varepsilon,\alpha) = \frac{den(p,\varepsilon,\alpha) - Card(N(p,\alpha \cdot \varepsilon))}{den(p,\alpha,\varepsilon)} = 1 - \frac{Card(N(p,\alpha \cdot \varepsilon))}{den(p,\alpha,\varepsilon)}$$

- σMDEF(p,ε,α) is the normalized standard deviation of the densities of all points from $N(p,\varepsilon)$
- Properties
  - MDEF = 0 for points within a cluster
  - MDEF > 0 for outliers or MDEF > 3·σMDEF => outlier

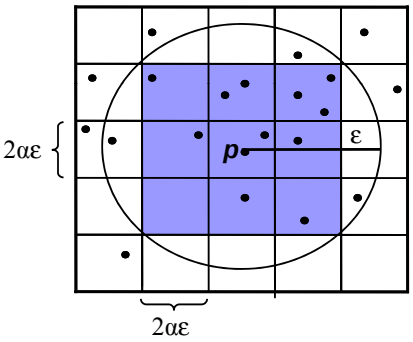
6.6 Density-based Approaches

- Features
  - Parameters ε and α are automatically determined
  - In fact, all possible values for ε are tested
  - LOCI plot displays for a given point p the following values w.r.t. ε
    - $Card(N(p, \alpha \cdot \varepsilon))$
    - $den(p, \varepsilon, \alpha)$ with a border of $\pm 3 \cdot \sigma den(p, \varepsilon, \alpha)$



6.6 Density-based Approaches

- Algorithms
  - Exact solution is rather expensive (compute MDEF values for all possible ε values)
  - aLOCI: fast, approximate solution
    - Discretize data space using a grid with side length 2αε
    - Approximate range queries trough grid cells
    - ε - neighborhood of point p: ζ(p,ε)
      all cells that are completely covered by ε-sphere around *p*
    - Then,



$$Card(N(q,\alpha\cdot\varepsilon)) = \frac{\sum_{c_j\in\zeta(p,\varepsilon)} c_j^{\,2}}{\sum_{c_j\in\zeta(p,\varepsilon)} c_j}$$

where c_j is the object count the corresponding cell

    - Since different ε values are needed, different grids are constructed with varying resolution
    - These different grids can be managed efficiently using a Quad-tree

6.6 Density-based Approaches

- Discussion
  - Exponential runtime w.r.t. data dimensionality
  - Output:
    - Label: if MDEF of a point > 3·σMDEF then this point is marked as outlier
    - LOCI plot
      - » At which resolution is a point an outlier (if any)
      - » Additional information such as diameter of clusters, distances to clusters, etc.
  - All interesting resolutions, i.e., possible values for ε, (from local to global) are tested

6 Outlier Detection

Übersicht

Statistisches Modell

Modellierung durch räumliche Nähe

Anpassung verschiedener Modelle an spezielles Problem

6.7 High-Dimensional Approaches

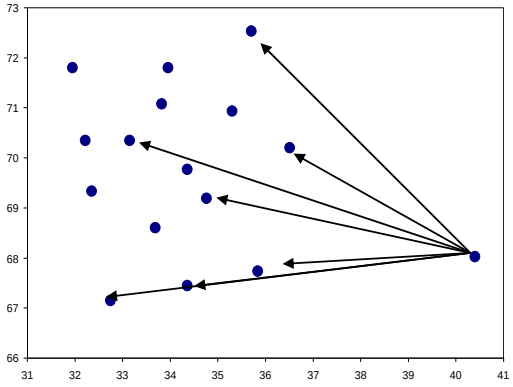
Challenges

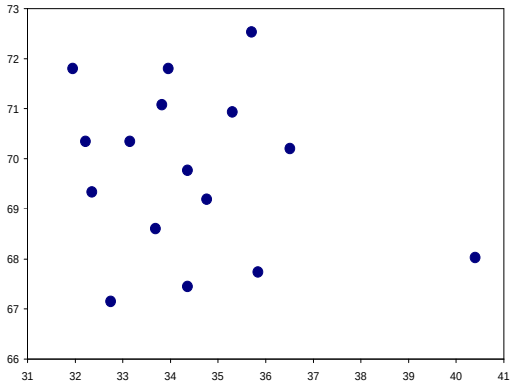
- Curse of dimensionality
  - Relative contrast between distances decreases with increasing dimensionality
  - Data are very sparse, almost all points are outliers
  - Concept of neighborhood becomes meaningless

- Solutions
  - Use more robust distance functions and find full-dimensional outliers
  - Find outliers in projections (subspaces) of the original feature space

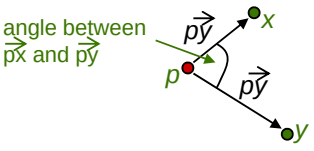
ABOD – angle-based outlier detection [Kriegel et al. 2008]

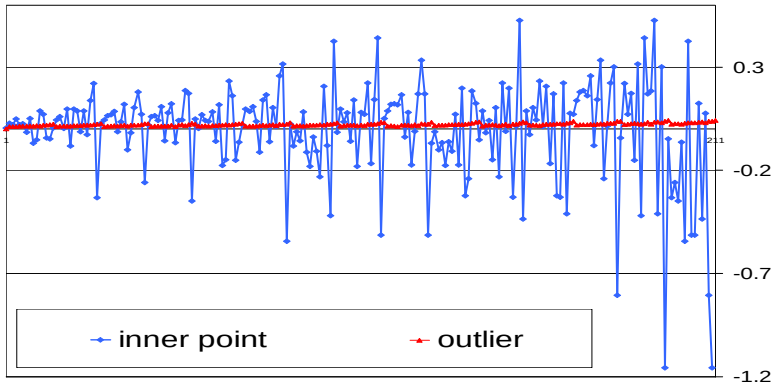
- Rational
  - Angles are more stable than distances in high dimensional spaces (cf. e.g. the popularity of cosine-based similarity measures for text data)
  - Object o is an outlier if most other objects are located in similar directions
  - Object o is no outlier if many other objects are located in varying directions





- Basic assumption
  - Outliers are at the border of the data distribution
  - Normal points are in the center of the data distribution
- Model
  - Consider for a given point p the angle between $\overrightarrow{px}$ and $\overrightarrow{py}$ for any two x,y from the database
  - Consider the spectrum of all these angles
  - The variance of this spectrum is a score for the outlierness of a point





6.7 High-Dimensional Approaches

- Model (cont.)
  - Measure the variance of the angle spectrum
  - Weighted inversely by the corresponding distances (if both points are far away, the angle has less impact on the overall variance)

$$ABOF(p) = \underset{x,y \in DB}{VAR}\left(\frac{\left\langle \overrightarrow{xp}, \overrightarrow{yp} \right\rangle}{\left\| \overrightarrow{xp} \right\|^2 \cdot \left\| \overrightarrow{yp} \right\|^2} \right)$$

  - Properties
    - Small ABOF => outlier
    - High ABOF => no outlier

6.7 High-Dimensional Approaches

- Algorithms
  - Naïve algorithm is in $O(n^3)$
  - Approximate algorithm based on random sampling for mining top-n outliers
    - Do not consider all pairs of other points x,y in the database to compute the angles
    - Compute ABOF based on samples => lower bound of the real ABOF
    - Filter out points that have a high lower bound
    - Refine (compute the exact ABOF value) only for a small number of points
- Discussion
  - Global approach to outlier detection
  - Outputs an outlier score

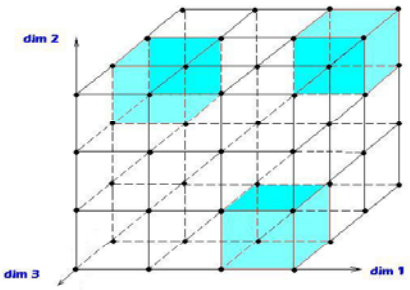
Grid-based subspace outlier detection [Aggarwal and Yu 2000]

- Model
  - Partition data space by an equi-depth grid (Φ = number of cells in each dimension)
  - Sparsity coefficient *S(C)* for a *k*-dimensional grid cell *C*

$$S(C) = \frac{count(C) - n \cdot (1/\Phi)^k}{\sqrt{n \cdot (1/\Phi)^k \cdot (1 - (1/\Phi)^k)}}$$

where *count*(*C*) is the number of data objects in C

  - *S(C)* < 0 => *count*(*C*) is lower than expected
  - Outliers are those objects that are located in lower-dimensional cells with negative sparsity coefficient



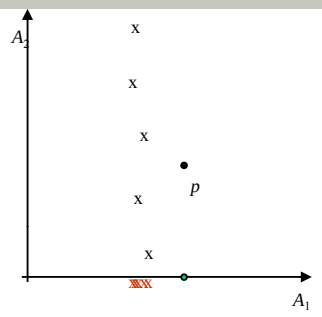
$\Phi = 3$

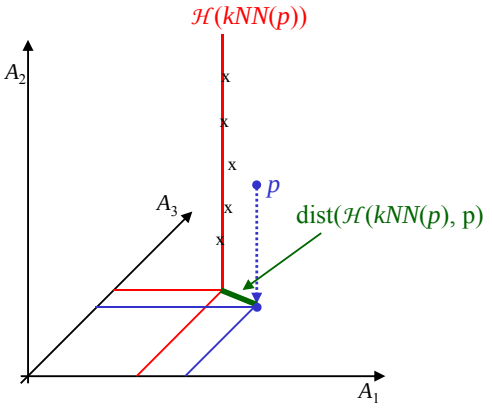
- Algorithm
  - Find the *m* grid cells (projections) with the lowest sparsity coefficients
  - Brute-force algorithm is in $O(\Phi^d)$
  - Evolutionary algorithm (input: *m* and the dimensionality of the cells)

- Discussion
  - Results need not be the points from the optimal cells
  - Very coarse model (all objects that are in cell with less points than to be expected)
  - Quality depends on grid resolution and grid position
  - Outputs a labeling
  - Implements a global approach (key criterion: globally expected number of points within a cell)

SOD – subspace outlier degree [Kriegel et al. 2009]

- Motivation
  - Outliers may be visible only in subspaces of the original data
- Model
  - Compute the subspace in which the *k*NNs of a point *p* minimize the variance
  - Compute the hyperplane $\mathcal{H}(kNN(p))$ that is orthogonal to that subspace
  - Take the distance of *p* to the hyperplane as measure for its "outlierness"





- Discussion
  - Assumes that *k*NNs of outliers have a lower-dimensional projection with small variance
  - Resolution is local (can be adjusted by the user via the parameter *k*)
  - Output is a scoring (SOD value)

6.8 Summary

Summary

- Different models are based on different assumptions to model outliers
- Different models provide different types of output (labeling/scoring)
- Different models consider outlier at different resolutions (global/local)
- Thus, different models will produce different results
- A thorough and comprehensive comparison between different models and approaches is still missing

6.8 Summary

Outlook

- Experimental evaluation of different approaches to understand and compare differences and common properties
- A first step towards unification of the diverse approaches: providing density-based outlier scores as probability values [Kriegel et al. 2009a]: judging the deviation of the outlier score from the expected value
- Visualization [Achtert et al. 2010]
- New models?
- Performance issues
- Complex data types
- High-dimensional data
- …
- UND VOR ALLEM:

  Viele interessante Bachelor-/Projekt-/Master-/Diplomarbeiten ☺

Literature

Elke Achtert, Hans-Peter Kriegel, Lisa Reichert, Erich Schubert, Remigius Wojdanowski, Arthur Zimek 2010. Visual Evaluation of Outlier Detection Models. In Proc. International Conference on Database Systems for Advanced Applications (DASFAA), Tsukuba, Japan.

Aggarwal, C.C. and Yu, P.S. 2000. Outlier detection for high dimensional data. In Proc. ACM SIGMOD Int. Conf. on Management of Data (SIGMOD), Dallas, TX.

Angiulli, F. and Pizzuti, C. 2002. Fast outlier detection in high dimensional spaces. In Proc. European Conf. on Principles of Knowledge Discovery and Data Mining, Helsinki, Finland.

Arning, A., Agrawal, R., and Raghavan, P. 1996. A linear method for deviation detection in large databases. In Proc. Int. Conf. on Knowledge Discovery and Data Mining (KDD), Portland, OR.

Barnett, V. 1978. The study of outliers: purpose and model. Applied Statistics, 27(3), 242–250.

Bay, S.D. and Schwabacher, M. 2003. Mining distance-based outliers in near linear time with randomization and a simple pruning rule. In Proc. Int. Conf. on Knowledge Discovery and Data Mining (KDD), Washington, DC.

Breunig, M.M., Kriegel, H.-P., Ng, R.T., and Sander, J. 1999. OPTICS-OF: identifying local outliers. In Proc. European Conf. on Principles of Data Mining and Knowledge Discovery (PKDD), Prague, Czech Republic.

Breunig, M.M., Kriegel, H.-P., Ng, R.T., and Sander, J. 2000. LOF: identifying density-based local outliers. In Proc. ACM SIGMOD Int. Conf. on Management of Data (SIGMOD), Dallas, TX.

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Literature

LMU

Ester, M., Kriegel, H.-P., Sander, J., and Xu, X. 1996. A density-based algorithm for discovering clusters in large spatial databases with noise. In Proc. Int. Conf. on Knowledge Discovery and Data Mining (KDD), Portland, OR.

Fan, H., Zaïane, O., Foss, A., and Wu, J. 2006. A nonparametric outlier detection for efficiently discovering top-n outliers from engineering data. In Proc. Pacific-Asia Conf. on Knowledge Discovery and Data Mining (PAKDD), Singapore.

Ghoting, A., Parthasarathy, S., and Otey, M. 2006. Fast mining of distance-based outliers in high dimensional spaces. In Proc. SIAM Int. Conf. on Data Mining (SDM), Bethesda, ML.

Hautamaki, V., Karkkainen, I., and Franti, P. 2004. Outlier detection using k-nearest neighbour graph. In Proc. IEEE Int. Conf. on Pattern Recognition (ICPR), Cambridge, UK.

Hawkins, D. 1980. Identification of Outliers. Chapman and Hall.

Jin, W., Tung, A., and Han, J. 2001. Mining top-*n* local outliers in large databases. In Proc. ACM SIGKDD Int. Conf. on Knowledge Discovery and Data Mining (SIGKDD), San Francisco, CA.

Jin, W., Tung, A., Han, J., and Wang, W. 2006. Ranking outliers using symmetric neighborhood relationship. In Proc. Pacific-Asia Conf. on Knowledge Discovery and Data Mining (PAKDD), Singapore.

Johnson, T., Kwok, I., and Ng, R.T. 1998. Fast computation of 2-dimensional depth contours. In Proc. Int. Conf. on Knowledge Discovery and Data Mining (KDD), New York, NY.

Knorr, E.M. and Ng, R.T. 1997. A unified approach for mining outliers. In Proc. Conf. of the Centre for Advanced Studies on Collaborative Research (CASCON), Toronto, Canada.

Literature

Knorr, E.M. and NG, R.T. 1998. Algorithms for mining distance-based outliers in large datasets. In Proc. Int. Conf. on Very Large Data Bases (VLDB), New York, NY.

Knorr, E.M. and Ng, R.T. 1999. Finding intensional knowledge of distance-based outliers. In Proc. Int. Conf. on Very Large Data Bases (VLDB), Edinburgh, Scotland.

Kriegel, H.-P., Kröger, P., Schubert, E., and Zimek, A. 2009. Outlier detection in axis-parallel subspaces of high dimensional data. In Proc. Pacific-Asia Conf. on Knowledge Discovery and Data Mining (PAKDD), Bangkok, Thailand.

Kriegel, H.-P., Kröger, P., Schubert, E., and Zimek, A. 2009a. LoOP: Local Outlier Probabilities. In Proc. ACM Conference on Information and Knowledge Management (CIKM), Hong Kong, China.

Kriegel, H.-P., Schubert, M., and Zimek, A. 2008. Angle-based outlier detection, In Proc. ACM SIGKDD Int. Conf. on Knowledge Discovery and Data Mining (SIGKDD), Las Vegas, NV.

McCallum, A., Nigam, K., and Ungar, L.H. 2000. Efficient clustering of high-dimensional data sets with application to reference matching. In Proc. ACM SIGKDD Int. Conf. on Knowledge Discovery and Data Mining (SIGKDD), Boston, MA.

Papadimitriou, S., Kitagawa, H., Gibbons, P., and Faloutsos, C. 2003. LOCI: Fast outlier detection using the local correlation integral. In Proc. IEEE Int. Conf. on Data Engineering (ICDE), Hong Kong, China.

Pei, Y., Zaiane, O., and Gao, Y. 2006. An efficient reference-based approach to outlier detection in large datasets. In Proc. 6th Int. Conf. on Data Mining (ICDM), Hong Kong, China.

Preparata, F. and Shamos, M. 1988. Computational Geometry: an Introduction. Springer Verlag.

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Literature

LMU

Ramaswamy, S. Rastogi, R. and Shim, K. 2000. Efficient algorithms for mining outliers from large data sets. In Proc. ACM SIGMOD Int. Conf. on Management of Data (SIGMOD), Dallas, TX.

Rousseeuw, P.J. and Leroy, A.M. 1987. Robust Regression and Outlier Detection. John Wiley.

Ruts, I. and Rousseeuw, P.J. 1996. Computing depth contours of bivariate point clouds. Computational Statistics and Data Analysis, 23, 153–168.

Tao Y., Xiao, X. and Zhou, S. 2006. Mining distance-based outliers from large databases in any metric space. In Proc. ACM SIGKDD Int. Conf. on Knowledge Discovery and Data Mining (SIGKDD), New York, NY.

Tan, P.-N., Steinbach, M., and Kumar, V. 2006. Introduction to Data Mining. Addison Wesley.

Tang, J., Chen, Z., Fu, A.W.-C., and Cheung, D.W. 2002. Enhancing effectiveness of outlier detections for low density patterns. In Proc. Pacific-Asia Conf. on Knowledge Discovery and Data Mining (PAKDD), Taipei, Taiwan.

Tukey, J. 1977. Exploratory Data Analysis. Addison-Wesley.

Zhang, T., Ramakrishnan, R., Livny, M. 1996. BIRCH: an efficient data clustering method for very large databases. In Proc. ACM SIGMOD Int. Conf. on Management of Data (SIGMOD), Montreal, Canada.