Exercise 3-1  Frequent Itemsets
The Apriori-algorithm makes use of prior knowledge of subset support properties. Let $I$ be the set of all items. Give proofs or counterexamples for the following claims:

(a) Let $S \subseteq I$ be a frequent itemset. Then every non-empty subset $S' \subseteq S$ must also be frequent.

(b) Let $S \subseteq I$ be an arbitrary itemset. Then $support(S') \geq support(S)$ holds for any non-empty subset $S' \subseteq S$.

Exercise 3-2  Frequent Itemset Mining
Let $D$ be a database that contains the following four transactions.

<table>
<thead>
<tr>
<th>TID</th>
<th>items_bought</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>{K, A, D, B}</td>
</tr>
<tr>
<td>T2</td>
<td>{D, A, C, E, B}</td>
</tr>
<tr>
<td>T3</td>
<td>{C, A, B, E}</td>
</tr>
<tr>
<td>T4</td>
<td>{B, A, D}</td>
</tr>
</tbody>
</table>

In addition let $min\_sup = 60\%$.

(a) Find all frequent itemsets using the Apriori algorithm.

(b) Find all frequent itemsets using the FP-growth algorithm.

(c) Determine all closed and maximal frequent itemsets.

Exercise 3-3  Association Rule Mining
After frequent itemset mining, association rules can be extracted as follows: For each frequent itemset $X$ and every non-empty subset $Y \subset X$, generate a rule $Y \Rightarrow X \setminus Y$ if it fulfills the minimum confidence property.

(a) Proof the following anti-monotonicity lemma for strong association rules:

Let $X$ be a frequent itemset and $Y \subset X$. If $Y \Rightarrow X \setminus Y$ is a strong association rule, then $Y' \Rightarrow X \setminus Y'$ is also a strong association rule for every $Y \subseteq Y'$.

(b) Extract all strong association rules from the database $D$ provided in the previous exercise with a minimum confidence of $min\_conf = 80\%$. Which candidate rules can be pruned based on anti-monotonicity?