Knowledge Discovery in Databases
SS 2016

Chapter 3: Frequent Itemset Mining

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Chapter 3: Frequent Itemset Mining

1) Introduction
   – Transaction databases, market basket data analysis
2) Mining Frequent Itemsets
   – Apriori algorithm, hash trees, FP-tree
3) Simple Association Rules
   – Basic notions, rule generation, interestingness measures
4) Further Topics
5) Extensions and Summary

What is Frequent Itemset Mining?

Frequent Itemset Mining:

Finding frequent patterns, associations, correlations, or causal structures among sets of items or objects in transaction databases, relational databases, and other information repositories.

- Given:
  - A set of items $I = \{i_1, i_2, \dots, i_m\}$
  - A database of transactions D, where a transaction $T \subseteq I$ is a set of items
- Task 1: find all subsets of items that occur together in many transactions.
  - E.g.: 85% of transactions contain the itemset {milk, bread, butter}
- Task 2: find all rules that correlate the presence of one set of items with that of another set of items in the transaction database.
  - E.g.: 98% of people buying tires and auto accessories also get automotive service done
- Applications: Basket data analysis, cross-marketing, catalog design, loss-leader analysis, clustering, classification, recommendation systems, etc.

Example: Basket Data Analysis

- Transaction database

  D= {{butter, bread, milk, sugar};
  {butter, flour, milk, sugar};
  {butter, eggs, milk, salt};
  {eggs};
  {butter, flour, milk, salt, sugar}}

- Question of interest:
  - Which items are bought together frequently?

items	frequency
{butter}	4
{milk}	4
{butter, milk}	4
{sugar}	3
{butter, sugar}	3
{milk, sugar}	3
{butter, milk, sugar}	3
{eggs}	2
…	

- Applications
  - Improved store layout
  - Cross marketing
  - Focused attached mailings / add-on sales
  - $* \Rightarrow$ *Maintenance Agreement*
    (What the store should do to boost Maintenance Agreement sales)
  - *Home Electronics* $\Rightarrow *$ (What other products should the store stock up?)

Chapter 3: Frequent Itemset Mining

Mining Frequent Itemsets: Basic Notions

- *Items* $I = \{i_1, i_2, \ldots, i_m\}$: a set of literals (denoting items)
- *Itemset* X: Set of items $X \subseteq I$
- *Database* D: Set of *transactions* T, each transaction is a set of items $\mathrm{T} \subseteq I$
- Transaction T *contains* an itemset X: $X \subseteq T$
- The items in transactions and itemsets are sorted lexicographically:
  - itemset $X = (x_1, x_2, \ldots, x_k)$, where $x_1 \leq x_2 \leq \ldots \leq x_k$
- *Length* of an itemset: number of elements in the itemset
- *k-itemset:* itemset of length *k*
- The *support* of an itemset X is defined as: $support(X) = |\{T \in D | X \subseteq T\}|$
- *Frequent itemset: an itemset* X is called frequent for database D iff it is contained in more than $minSup$ many transactions: $support(X) \geq minSup$

- Goal 1: Given a database $D$and a threshold $minSup$, find all frequent itemsets $\mathrm{X} \in Pot(I)$.

Mining Frequent Itemsets: Basic Idea

- Naïve Algorithm
  - count the frequency of all possible subsets of I in the database

  → *too expensive* since there are 2^m such itemsets for $|I| = m$ items

  cardinality of power set

- The *Apriori* principle (anti-monotonicity):

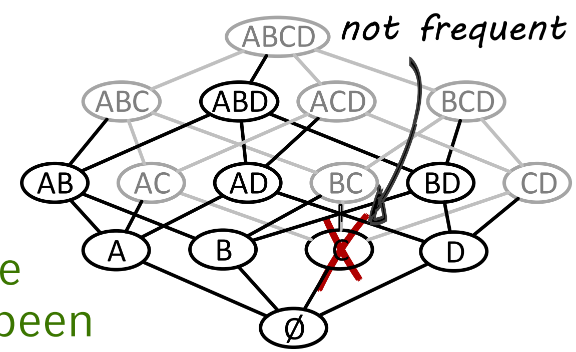
  Any non-empty subset of a frequent itemset is frequent, too!

  $A \subseteq I \text{ with support}(A) \geq \text{minSup} \Rightarrow \forall A' \subset A \wedge A' \neq \emptyset: \text{support}(A') \geq \text{minSup}$

  Any superset of a non-frequent itemset is non-frequent, too!

  $A \subseteq I \text{ with support}(A) < \text{minSup} \Rightarrow \forall A' \supset A: \text{support}(A') < \text{minSup}$

- Method based on the apriori principle
  - First count the 1-itemsets, then the 2-itemsets, then the 3-itemsets, and so on
  - When counting (k+1)-itemsets, only consider those (k+1)-itemsets where all subsets of length k have been determined as frequent in the previous step



The Apriori Algorithm

variable C_k: candidate itemsets of size k
variable L_k: frequent itemsets of size k

L_1 = {frequent items}
for (k = 1; L_k !=∅; k++) **do begin**

produce candidates

// JOIN STEP: join L_k with itself to produce C_{k+1}
// PRUNE STEP: discard (k+1)-itemsets from C_{k+1} that contain non-frequent k-itemsets as subsets
C_{k+1} = candidates generated from L_k

prove candidates

for each transaction t in database do
Increment the count of all candidates in C_{k+1} that are contained in t
L_{k+1} = candidates in C_{k+1} with min_support
return $\cup_k L_k$

Generating Candidates (Join Step)

- Requirements for set of all candidate $(k+1)$-itemsets C_{k+1}
  - *Completeness*:
    Must contain all frequent $(k+1)$-itemsets (superset property $C_{k+1} \supseteq L_{k+1}$)
  - *Selectiveness*:
    Significantly smaller than the set of all $(k+1)$-subsets
  - Suppose the items are sorted by any order (e.g., lexicograph.)
- Step 1: Joining ($C_{k+1} = L_k \bowtie L_k$)
  - Consider frequent k-itemsets p and q
  - p and q are joined if they share the same first $k-1$ items

insert into C_{k+1}

select $p.i_1, p.i_2, \ldots, p.i_{k-1}, p.i_k, q.i_k$

from $L_k : p, L_k : q$

where $p.i_1 = q.i_1, \ldots, p.i_{k-1} = q.i_{k-1}, p.i_k < q.i_k$

$p \in L_{k=3}$ (A, C, F)

(A, C, F, G) $\in C_{k+1=4}$

$q \in L_{k=3}$ (A, C, G)

Generating Candidates (Prune Step)

- Step 2: Pruning ($L_{k+1} = \{X \in C_{k+1} | support(X) \geq minSup\}$)
  - Naïve: Check support of every itemset in C_{k+1} ← inefficient for huge C_{k+1}
  - Instead, apply Apriori principle first: Remove candidate *(k+1)* -itemsets which contain a non-frequent *k* -subset *s*, i.e., s $\notin L_k$

    forall *itemsets c* ***in*** C_{k+1} **do**
        forall *k-subsets s* ***of*** *c* **do**
            if *(s is not in* L_k*)* **then delete** *c* **from** C_{k+1}

- Example 1
  - L_3 = {(ACF), (ACG), (AFG), (AFH), (CFG)}
  - Candidates after the join step: {(ACFG), (AFGH)}
  - In the pruning step: delete (AFGH) because (FGH) $\notin L_3$, i.e., (FGH) is not a frequent 3-itemset; also (AGH) $\notin L_3$

  → C_4 = {(ACFG)} → check the support to generate L_4

Apriori Algorithm – Full Example

minSup=0.5

database D

TID	items
100	1 3 4 6
200	2 3 5
300	1 2 3 5
400	1 5 6

scan D →

C_1

itemset	count
{1}	3
{2}	2
{3}	3
{4}	1
{5}	3
{6}	2

→

L_1

itemset	count
{1}	3
{2}	2
{3}	3
{5}	3
{6}	2

$L_1 \bowtie L_1$

C_2

itemset
{1 2}
{1 3}
{1 5}
{1 6}
{2 3}
{2 5}
{2 6}
{3 5}
{3 6}
{5 6}

prune C_1 →

C_2

itemset
{1 2}
{1 3}
{1 5}
{1 6}
{2 3}
{2 5}
{2 6}
{3 5}
{3 6}
{5 6}

scan D →

C_2

itemset	count
{1 2}	1
{1 3}	2
{1 5}	2
{1 6}	2
{2 3}	2
{2 5}	2
{2 6}	0
{3 5}	2
{3 6}	1
{5 6}	1

→

L_2

itemset	count
{1 3}	2
{1 5}	2
{1 6}	2
{2 3}	2
{2 5}	2
{3 5}	2

$L_2 \bowtie L_2$

C_3

itemset
{1 3 5}
{1 3 6}
{1 5 6}
{2 3 5}

prune C_2 →

C_3

itemset
{1 3 5}
{1 3 6} ✗
{1 5 6} ✗
{2 3 5}

scan D →

C_3

itemset	count
{1 3 5}	1
{2 3 5}	2

→

L_3

itemset	count
{2 3 5}	2

$L_3 \bowtie L_3$

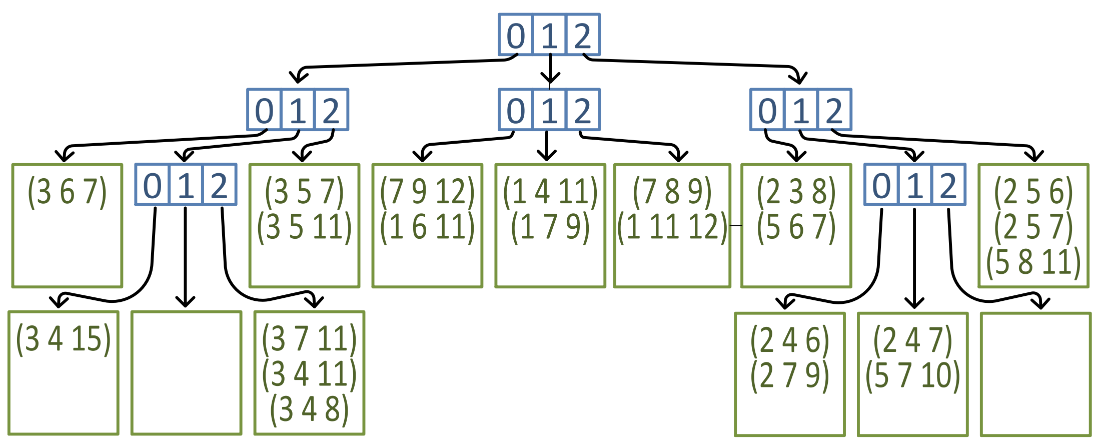
C_4 is empty

How to Count Supports of Candidates?

- Why is counting supports of candidates a problem?
  - The total number of candidates can be very huge
  - One transaction may contain many candidates
- Method: Hash-Tree
  - Candidate itemsets are stored in a hash-tree
  - Leaf nodes of hash-tree contain lists of itemsets and their support (i.e., counts)
  - Interior nodes contain hash tables
  - Subset function finds all the candidates contained in a transaction

e.g. for 3-Itemsets

$h(K) = K \bmod 3$

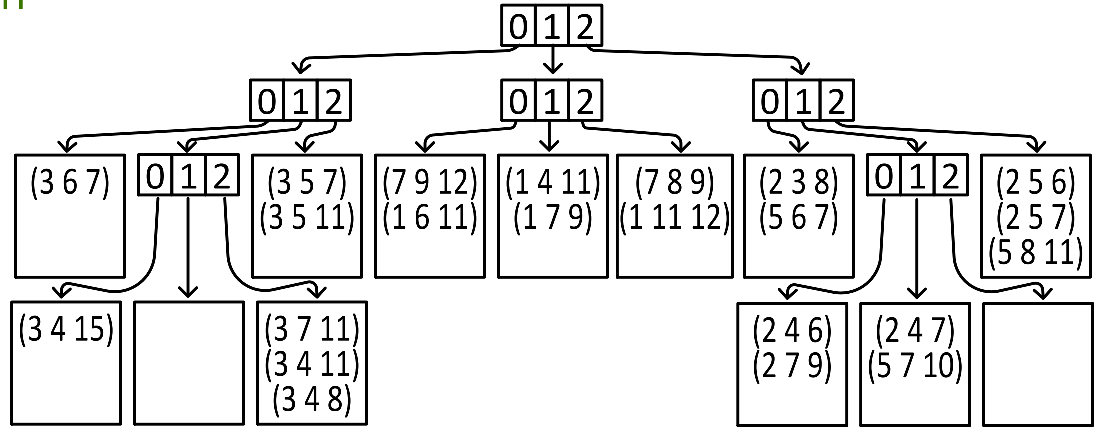


Hash-Tree – Construction

- Searching for an itemset
  - Start at the root (level 1)
  - At level *d:* apply the hash function *h* to the *d*-th item in the itemset
- Insertion of an itemset
  - search for the corresponding leaf node, and insert the itemset into that leaf
  - if an overflow occurs:
    - Transform the leaf node into an internal node
    - Distribute the entries to the new leaf nodes according to the hash function

for 3-Itemsets

$h(K) = K \bmod 3$



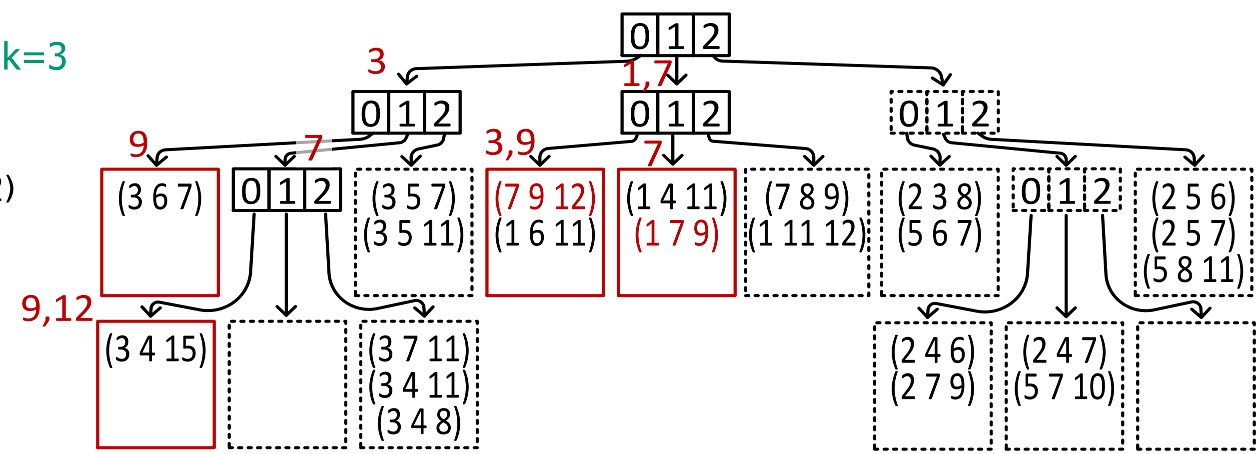
Hash-Tree – Counting

- Search all candidate itemsets contained in a transaction $T = (t_1\ t_2\ \ldots\ t_n)$ for a current itemset length of k
- At the root
  - Determine the hash values for each item $t_1\ t_2\ \ldots\ t_{n-k+1}$ in T
  - Continue the search in the resulting child nodes
- At an internal node at level d (reached after hashing of item t_i)
  - Determine the hash values and continue the search for each item t_j with $i < j \leq n - k + d$
- At a leaf node
  - Check whether the itemsets in the leaf node are contained in transaction T

in our example n=5 and k=3

$h(K) = K \bmod 3$

Transaction (1, 3, 7, 9, 12)



Tested leaf nodes

Pruned subtrees

Is Apriori Fast Enough? — Performance Bottlenecks

- The core of the Apriori algorithm:
  - Use frequent (k – 1)-itemsets to generate candidate frequent k-itemsets
  - Use database scan and pattern matching to collect counts for the candidate itemsets
- The bottleneck of *Apriori*: candidate generation
  - Huge candidate sets:
    - 10^4 frequent 1-itemsets will generate 10^7 candidate 2-itemsets
    - To discover a frequent pattern of size 100, e.g., $\{a_1, a_2, \ldots, a_{100}\}$, one needs to generate $2^{100} \approx 10^{30}$ candidates.
  - Multiple scans of database:
    - Needs n or n+1 scans, n is the length of the longest pattern

→ Is it possible to mine the complete set of frequent itemsets without candidate generation?

Mining Frequent Patterns Without Candidate Generation

- Compress a large database into a compact, *Frequent-Pattern tree* (*FP-tree*) structure
  - highly condensed, but complete for frequent pattern mining
  - avoid costly database scans
- Develop an efficient, FP-tree-based frequent pattern mining method
  - A divide-and-conquer methodology: decompose mining tasks into smaller ones
  - Avoid candidate generation: sub-database test only!
- Idea:
  - Compress database into FP-tree, retaining the itemset association information
  - Divide the compressed database into conditional databases, each associated with one frequent item and mine each such database separately.

Construct FP-tree from a Transaction DB

Steps for compressing the database into a FP-tree:

1. Scan DB once, find frequent 1-itemsets (single items)
2. Order frequent items in frequency descending order

TID	items bought
100	{*f, a, c, d, g, i, m, p*}
200	{*a, b, c, f, l, m, o*}
300	{*b, f, h, j, o*}
400	{*b, c, k, s, p*}
500	{*a, f, c, e, l, p, m, n*}

1&2

minSup=0.5

header table:

item	frequency
f	4
c	4
a	3
b	3
m	3
p	3

sort items in the order of descending support

Construct FP-tree from a Transaction DB

Steps for compressing the database into a FP-tree:

1. Scan DB once, find frequent 1-itemsets (single items)
2. Order frequent items in frequency descending order
3. Scan DB again, construct FP-tree starting with most frequent item per transaction

TID	items bought	(ordered) frequent items
100	{*f, a, c, d, g, i, m, p*}	{*f, c, a, m, p*}
200	{*a, b, c, f, l, m, o*}	{*f, c, a, b, m*}
300	{*b, f, h, j, o*}	{*f, b*}
400	{*b, c, k, s, p*}	{*c, b, p*}
500	{*a, f, c, e, l, p, m, n*}	{*f, c, a, m, p*}

for each transaction only keep its frequent items sorted in descending order of their frequencies

1&2

header table:

item	frequency
f	4
c	4
a	3
b	3
m	3
p	3

3a

for each transaction build a path in the FP-tree:

- If a path with common prefix exists: increment frequency of nodes on this path and append suffix
- Otherwise: create a new branch

Construct FP-tree from a Transaction DB

Steps for compressing the database into a FP-tree:

1. Scan DB once, find frequent 1-itemsets (single items)
2. Order frequent items in frequency descending order
3. Scan DB again, construct FP-tree starting with most frequent item per transaction

TID	items bought	(ordered) frequent items
100	{f, a, c, d, g, i, m, p}	{f, c, a, m, p}
200	{a, b, c, f, l, m, o}	{f, c, a, b, m}
300	{b, f, h, j, o}	{f, b}
400	{b, c, k, s, p}	{c, b, p}
500	{a, f, c, e, l, p, m, n}	{f, c, a, m, p}

1&2

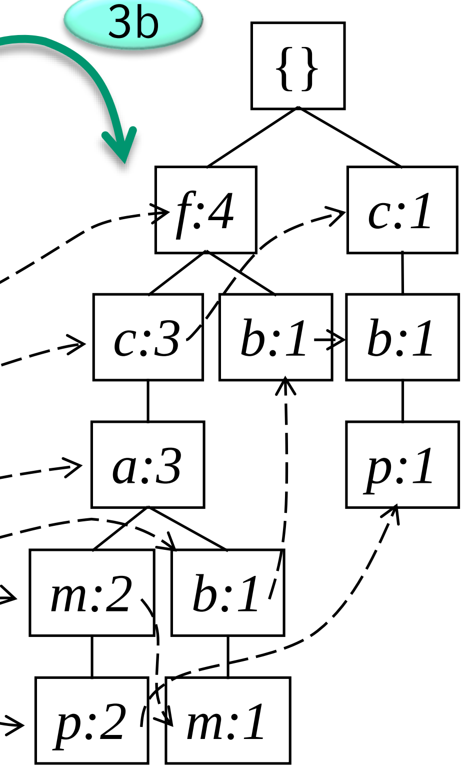
3a

3b

header table:

item	frequency	head
f	4	•
c	4	•
a	3	•
b	3	•
m	3	•
p	3	•

header table references the occurrences of the frequent items in the FP-tree



Benefits of the FP-tree Structure

- Completeness:
  - never breaks a long pattern of any transaction
  - preserves complete information for frequent pattern mining
- Compactness
  - reduce irrelevant information—infrequent items are gone
  - frequency descending ordering: more frequent items are more likely to be shared
  - never be larger than the original database (if not count node-links and counts)
  - Experiments demonstrate compression ratios over 100

Mining Frequent Patterns Using FP-tree

- General idea (divide-and-conquer)
  - Recursively grow frequent pattern path using the FP-tree
- Method
  - For each item, construct its conditional pattern-base (*prefix paths*), and then its conditional FP-tree
  - Repeat the process on each newly created conditional FP-tree …
  - …until the resulting FP-tree is empty, or it contains only one path (single path will generate all the combinations of its sub-paths, each of which is a frequent pattern)

Major Steps to Mine FP-tree

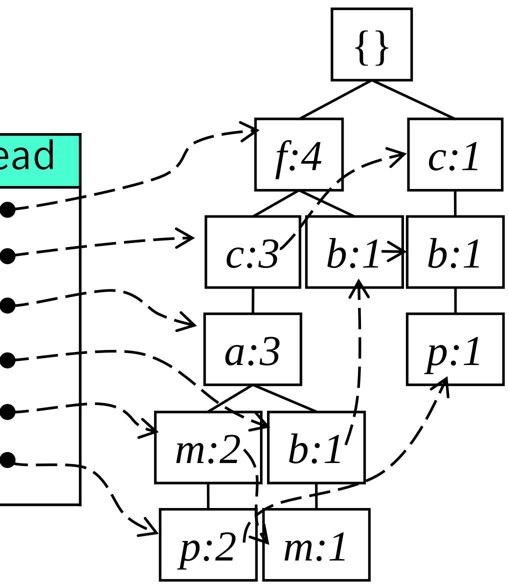
1) Construct conditional pattern base for each node in the FP-tree
2) Construct conditional FP-tree from each conditional pattern-base
3) Recursively mine conditional FP-trees and grow frequent patterns obtained so far
   – If the conditional FP-tree contains a single path, simply enumerate all the patterns

Major Steps to Mine FP-tree: Conditional Pattern Base

1) Construct conditional pattern base for each node in the FP-tree
   – Starting at the frequent header table in the FP-tree
   – Traverse FP-tree by following the link of each frequent item (dashed lines)
   – Accumulate all of transformed prefix paths of that item to form a conditional pattern base
      • For each item its prefixes are regarded as condition for it being a suffix. These prefixes form the conditional pattern base. The frequency of the prefixes can be read in the node of the item.

header table:

item	frequency	head
f	4	•
c	4	•
a	3	•
b	3	•
m	3	•
p	3	•



conditional pattern base:

item	cond. pattern base
f	*{}*
c	*f:3, {}*
a	*fc:3*
b	*fca:1, f:1, c:1*
m	*fca:2, fcab:1*
p	*fcam:2, cb:1*

Properties of FP-tree for Conditional Pattern Bases

- Node-link property
  - For any frequent item a_i, all the possible frequent patterns that contain a_i can be obtained by following a_i's node-links, starting from a_i's head in the FP-tree header
- Prefix path property
  - To calculate the frequent patterns for a node a_i in a path P, only the prefix sub-path of a_i in P needs to be accumulated, and its frequency count should carry the same count as node a_i.

Major Steps to Mine FP-tree: Conditional FP-tree

1) Construct conditional pattern base for each node in the FP-tree
2) Construct conditional FP-tree from each conditional pattern-base
   - The prefix paths of a suffix represent the conditional basis.
     →They can be regarded as transactions of a database.
   - Those prefix paths whose support ≥ minSup, induce a conditional FP-tree
   - For each pattern-base
     - Accumulate the count for each item in the base
     - Construct the FP-tree for the frequent items of the pattern base

conditional pattern base:

item	cond. pattern base
f	*{}*
c	*f:3*
a	*fc:3*
b	*fca:1, f:1, c:1*
m	*fca:2, fcab:1*
p	*fcam:2, cb:1*

e.g. item m

item	frequency
f	3
c	3
a	3
b	1 X

m-conditional FP-tree

{}|m
|
f:3
|
c:3
|
a:3

Major Steps to Mine FP-tree: Conditional FP-tree

1) Construct conditional pattern base for each node in the FP-tree
2) Construct conditional FP-tree from each conditional pattern-base

conditional pattern base:

item	cond. pattern base
f	*{}*
c	*f:3*
a	*fc:3*
b	*fca:1, f:1, c:1*
m	*fca:2, fcab:1*
p	*fcam:2, cb:1*

{}|f = {}

{}|c
|
f:3

{}|a
|
f:3
|
c:3

{}|b = {}

{}|m
|
f:3
|
c:3
|
a:3

{}|p
|
c:3

Major Steps to Mine FP-tree

1) Construct conditional pattern base for each node in the FP-tree
2) Construct conditional FP-tree from each conditional pattern-base
3) Recursively mine conditional FP-trees and grow frequent patterns obtained so far
   – If the conditional FP-tree contains a single path, simply enumerate all the patterns (enumerate all combinations of sub-paths)

example:
m-conditional FP-tree

{}|m
|
f:3
|
c:3
|
a:3

just a single path →

All frequent patterns concerning *m*

m,

fm, cm, am,

fcm, fam, cam,

fcam

DATABASE SYSTEMS GROUP

FP-tree: Full Example

LMU

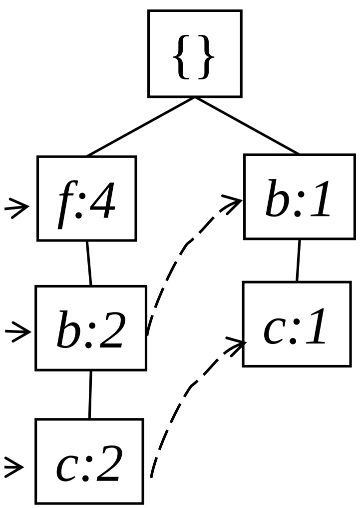
database:

TID	items bought	(ordered) frequent items
100	{*b, c, f*}	{*f, b, c*}
200	{*a, b, c*}	{*b, c*}
300	{*d, f*}	{*f*}
400	{*b, c, e, f*}	{*f, b, c*}
500	{*f, g*}	{*f*}

minSup=0.4

header table:

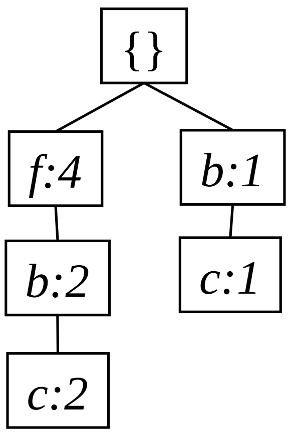
item	frequency	head
f	4	•
b	3	•
c	3	•



conditional pattern base:

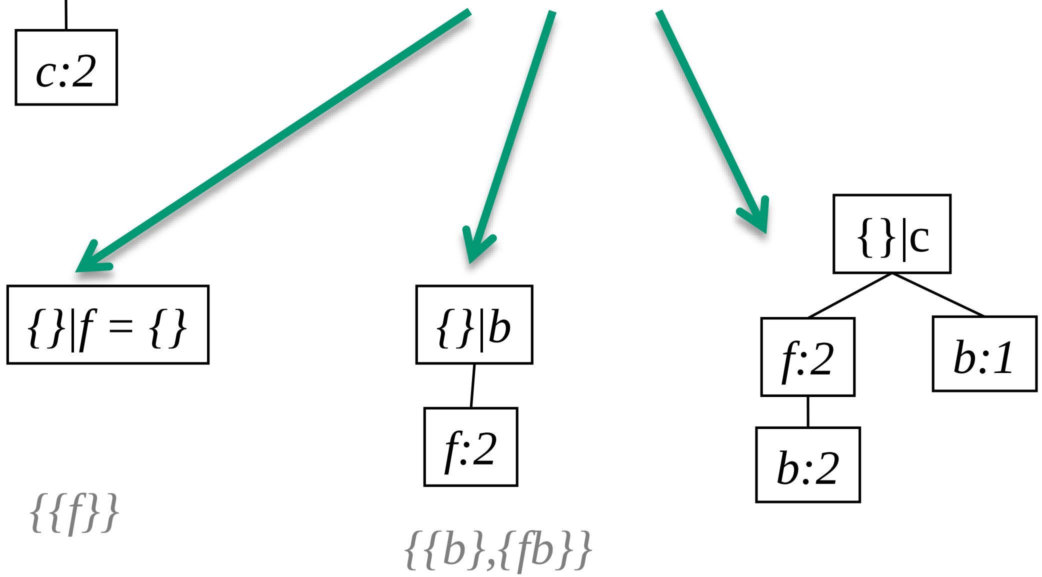
item	cond. pattern base
f	{}
b	*f*:2, {}
c	*fb*:2, *b*:1

FP-tree: Full Example



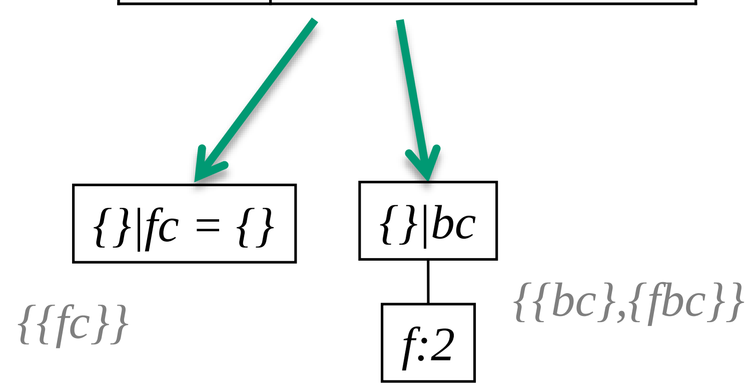
conditional pattern base 1:

item	cond. pattern base
f	{}
b	f:2
c	fb:2, b:1



conditional pattern base 2:

item	cond. pattern base
b	f:2
f	{}



Principles of Frequent Pattern Growth

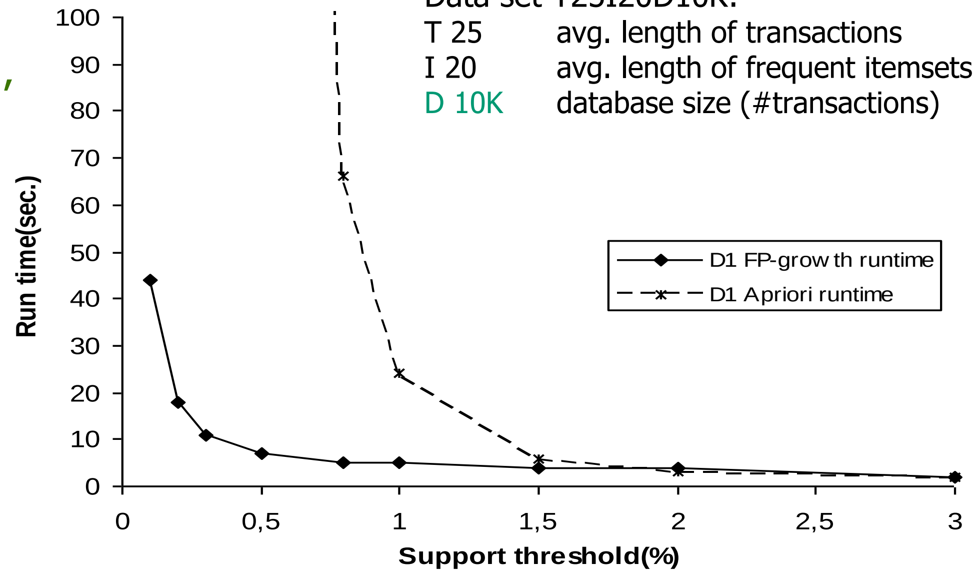
- Pattern growth property
  - Let α be a frequent itemset in DB, B be α's conditional pattern base, and β be an itemset in B. Then $\alpha \cup \beta$ is a frequent itemset in DB iff β is frequent in B.

- "*abcdef*" is a frequent pattern, if and only if
  - "*abcde*" is a frequent pattern, and
  - "*f*" is frequent in the set of transactions containing "*abcde*"

Why Is Frequent Pattern Growth Fast?

- Performance study in [Han, Pei&Yin ’00] shows
  - FP-growth is an order of magnitude faster than Apriori, and is also faster than tree-projection

Data set T25I20D10K:
T 25 avg. length of transactions
I 20 avg. length of frequent itemsets
D 10K database size (#transactions)



- Reasoning
  - No candidate generation, no candidate test
    - Apriori algorithm has to proceed breadth-first
  - Use compact data structure
  - Eliminate repeated database scan
  - Basic operation is counting and FP-tree building

Maximal or Closed Frequent Itemsets

- Big challenge: database contains potentially a huge number of frequent itemsets (especially if minSup is set too low).
  - A frequent itemset of length 100 contains 2^{100}-1 many frequent subsets
- *Closed frequent itemset*:
  An itemset X is *closed* in a data set D if there exists no proper super-itemset Y such that $support(X) = support(Y)$ in D.
  - The set of closed frequent itemsets contains complete information regarding its corresponding frequent itemsets.
- *Maximal frequent itemset*:
  An itemset X is *maximal* in a data set D if there exists no proper super-itemset Y such that $support(Y) \geq minSup$ in D.
  - The set of maximal itemsets does not contain the complete support information
  - More compact representation

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2) Mining Frequent Itemsets
   - Apriori algorithm, hash trees, FP-tree
3) Simple Association Rules
   - Basic notions, rule generation, interestingness measures
4) Further Topics
   - Hierarchical Association Rules
     - Motivation, notions, algorithms, interestingness
   - Quantitative Association Rules
     - Motivation, basic idea, partitioning numerical attributes, adaptation of apriori algorithm, interestingness
5) Extensions and Summary

Simple Association Rules: Introduction

- Transaction database:

  D= {{butter, bread, milk, sugar};
  {butter, flour, milk, sugar};
  {butter, eggs, milk, salt};
  {eggs};
  {butter, flour, milk, salt, sugar}}

- Frequent itemsets:

items	support
{butter}	4
{milk}	4
{butter, milk}	4
{sugar}	3
{butter, sugar}	3
{milk, sugar}	3
{butter, milk, sugar}	3

- Question of interest:
  - If milk and sugar are bought, will the customer always buy butter as well?
    $milk, sugar \Rightarrow butter$?
  - In this case, what would be the probability of buying butter?

Simple Association Rules: Basic Notions

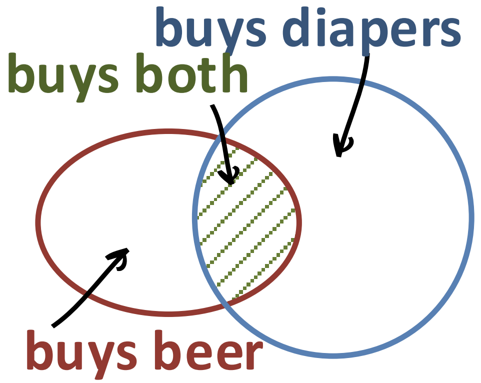
- *Items* $I = \{i_1, i_2, \ldots, i_m\}$: a set of literals (denoting items)
- *Itemset* X*:* Set of items $X \subseteq I$
- *Database* D: Set of *transactions* T, each transaction is a set of items $T \subseteq I$
- Transaction T *contains* an itemset X: $X \subseteq T$
- The items in transactions and itemsets are sorted lexicographically:
  - itemset $X = (x_1, x_2, \ldots, x_k)$, where $x_1 \leq x_{2 \leq} \ldots \leq x_k$
- *Length* of an itemset: cardinality of the itemset (*k-itemset*: itemset of length *k)*
- The *support* of an itemset X is defined as: $support(X) = |\{T \in D | X \subseteq T\}|$
- *Frequent itemset: an itemset* X is called frequent iff $support(X) \geq minSup$

- *Association rule:* An association rule is an implication of the form $X \Rightarrow Y$ where $X, Y \subseteq I$ are two itemsets with $X \cap Y = \emptyset$.

- Note: simply enumerating all possible association rules is not reasonable!
  → What are the interesting association rules w.r.t. D?

Interestingness of Association Rules

- *Interestingness of an association rule:*
  Quantify the interestingness of an association rule with respect to a transaction database D:
  - Support: frequency (probability) of the entire rule with respect to D
    $$support(X \Rightarrow Y) = P(X \cup Y) = \frac{|\{T \in D | X \cup Y \subseteq T\}|}{|D|} = support(X \cup Y)$$
    "probability that a transaction in D contains the itemset $X \cup Y$"
  - Confidence: indicates the strength of implication in the rule
    $$confidence(X \Rightarrow Y) = P(Y|X) = \frac{|\{T \in D | X \cup Y \subseteq T\}|}{|\{T \in D | X \subseteq T\}|} = \frac{support(X \cup Y)}{support(X)}$$
    "conditional probability that a transaction in D containing the itemset X also contains itemset Y"
  - Rule form: "$Body \Rightarrow Head\ [support, confidence]$"
- Association rule examples:
  - buys diapers ⇒ buys beers [0.5%, 60%]
  - major in CS ∧ takes DB ⇒ avg. grade A [1%, 75%]



Mining of Association Rules

- *Task of mining association rules:*
  Given a database D, determine all association rules having a $support \geq minSup$ and a $confidence \geq minConf$ (so-called *strong association rules*).
- Key steps of mining association rules:
  e.g. Apriori, FP-growth
  1) Find *frequent itemsets*, i.e., itemsets that have at least support $= minSup$
  2) Use the frequent itemsets to generate association rules
     - For each itemset X and every nonempty subset $Y \subset X$ generate rule $Y \Rightarrow (X - Y)$ if $minSup$ and $minConf$ are fulfilled
     - we have $2^{|X|} - 2$ many association rule candidates for each itemset X
- Example
  frequent itemsets

1-itemset	count	2-itemset	count	3-itemset	count
{A} {B} {C}	3 4 5	{A, B} {A, C} {B, C}	3 2 4	{A, B, C}	2

rule candidates: $A \Rightarrow B; B \Rightarrow A;\ A \Rightarrow C;\ C \Rightarrow A;\ B \Rightarrow C;\ C \Rightarrow B;$
$A,B \Rightarrow C; A,C \Rightarrow B; C,B \Rightarrow A; A \Rightarrow B,C; B \Rightarrow A,C; C \Rightarrow A,B$

Generating Rules from Frequent Itemsets

- For each frequent itemset X
  - For each nonempty subset Y of X, form a rule $\mathrm{Y} \Rightarrow (X - Y)$
  - Delete those rules that do not have minimum confidence
    Note: 1) support always exceeds $minSup$
    2) the support values of the frequent itemsets suffice to calculate the confidence
- Example: $X = \{A, B, C\}$, $minConf = 60\%$
  - conf (A ⇒ B) = 3/3; ✓
  - conf (B ⇒ A) = 3/4; ✓
  - conf (A ⇒ C) = 2/3; ✓
  - conf (C ⇒ A) = 2/5; ✗
  - conf (B ⇒ C) = 4/4; ✓
  - conf (C ⇒ B) = 4/5; ✓
  - conf (A ⇒ B, C) = 2/3; ✓ conf (B, C ⇒ A) = ½ ✗
  - conf (B ⇒ A, C) = 2/4; ✗ conf (A, C ⇒ B) = 1 ✓
  - conf (C ⇒ A, B) = 2/5; ✗ conf (A, B ⇒ C) = 2/3 ✓

itemset	count
{A}	3
{B}	4
{C}	5
{A, B}	3
{A, C}	2
{B, C}	4
{A, B, C}	2

- Exploit anti-monotonicity for generating candidates for strong association rules!

Interestingness Measurements

- *Objective* measures
  - Two popular measurements:
  - support and
  - confidence

- *Subjective* measures [Silberschatz & Tuzhilin, KDD95]
  - A rule (pattern) is interesting if it is
  - unexpected (surprising to the user) and/or
  - actionable (the user can do something with it)

Criticism to Support and Confidence

Example 1 [Aggarwal & Yu, PODS98]

- Among 5000 students
  - 3000 play basketball (=60%)
  - 3750 eat cereal (=75%)
  - 2000 both play basket ball and eat cereal (=40%)
- Rule *play basketball* $\Rightarrow$ *eat cereal* [40%, 66.7%] is misleading because the overall percentage of students eating cereal is 75% which is higher than 66.7%
- Rule *play basketball* $\Rightarrow$ *not eat cereal* [20%, 33.3%] is far more accurate, although with lower support and confidence
- Observation: *play basketball* and *eat cereal* are *negatively correlated*

➢ Not all strong association rules are interesting and some can be misleading.
→ augment the support and confidence values with interestingness measures such as the correlation $A \Rightarrow B\ [supp, conf, corr]$

Other Interestingness Measures: Correlation

- ***Lift*** is a simple correlation measure between two items *A* and *B:*

$$corr_{A,B} = \frac{P(A \cup B)}{P(A)P(B)} = \frac{P(B|A)}{P(B)} = \frac{conf(A \Rightarrow B)}{supp(B)}$$

! The two rules $A \Rightarrow B$ and $B \Rightarrow A$ have the same correlation coefficient.

- take both P(A) and P(B) in consideration

- $corr_{A,B} > 1$ the two items A and B are positively correlated
- $corr_{A,B} = 1$ there is no correlation between the two items A and B
- $corr_{A,B} < 1$ the two items A and B are negatively correlated

Other Interestingness Measures: Correlation

- Example 2:

X	1	1	1	1	0	0	0	0
Y	1	1	0	0	0	0	0	0
Z	0	1	1	1	1	1	1	1

- X and Y: positively correlated
- X and Z: negatively related
- support and confidence of X=>Z dominates
- but items X and Z are negatively correlated
- Items X and Y are positively correlated

rule	support	confidence	correlation
$X \Rightarrow Y$	25%	50%	2
$X \Rightarrow Z$	37.5%	75%	0.86
$Y \Rightarrow Z$	12.5%	50%	0.57

Chapter 3: Frequent Itemset Mining

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4) Further Topics
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   - Quantitative Association Rules
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5) Extensions and Summary

Hierarchical Association Rules: Motivation

- Problem of association rules in plain itemsets
  - *High minsup:* apriori finds only few rules
  - *Low minsup:* apriori finds unmanagably many rules
- Exploit item taxonomies (generalizations, *is-a* hierarchies) which exist in many applications

- New task: find all generalized association rules between generalized items → Body and Head of a rule may have items of any level of the hierarchy
- Generalized association rule: $X \Rightarrow Y$
  with $X, Y \subset I, X \cap Y = \emptyset$ and no item in Y is an ancestor of any item in X
  i.e., $jackets \Rightarrow clothes$ is essentially true

Hierarchical Association Rules: Motivating Example

- Examples

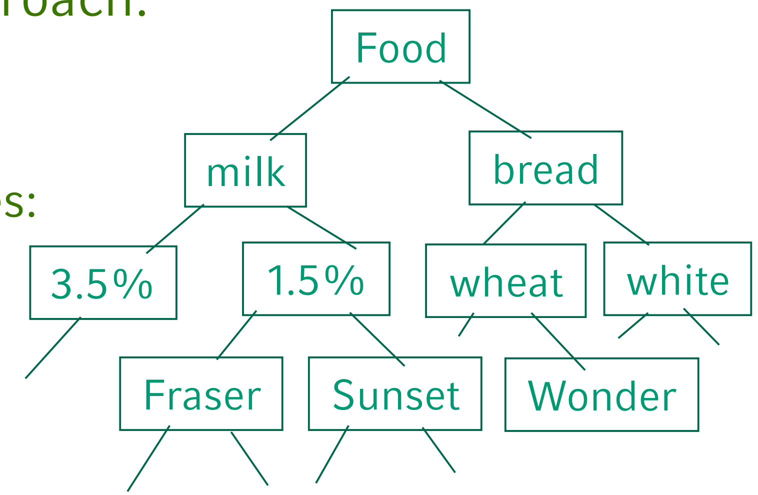
  Jeans $\Rightarrow$ boots
  jackets $\Rightarrow$ boots
  } Support < minSup

  Outerwear $\Rightarrow$ boots Support > minsup

- Characteristics
  - Support("outerwear $\Rightarrow$ boots") is not necessarily equal to the sum support("jackets $\Rightarrow$ boots") + support("jeans $\Rightarrow$ boots")
    e.g. if a transaction with jackets, jeans and boots exists
  - Support for sets of generalizations (e.g., product groups) is higher than support for sets of individual items
    If the support of rule "outerwear $\Rightarrow$ boots" exceeds minsup, then the support of rule "clothes $\Rightarrow$ boots" does, too

Mining Multi-Level Associations

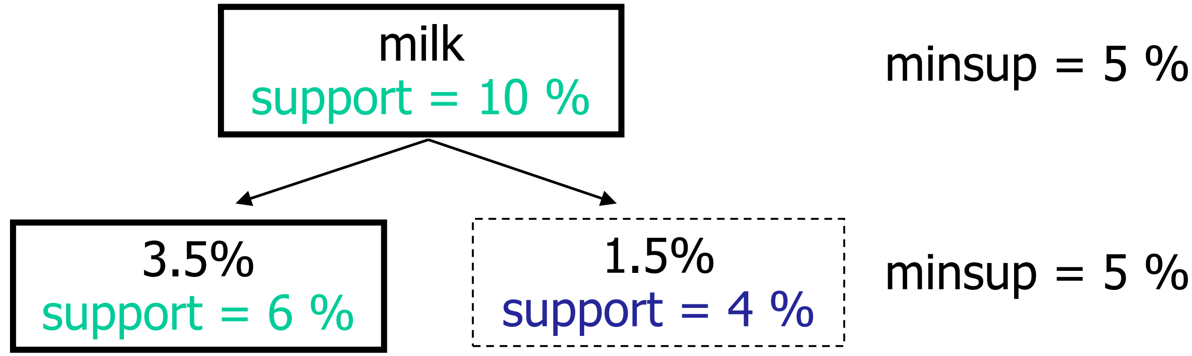
- A *top_down, progressive deepening* approach:
  - First find high-level strong rules:
    - *milk* ⇒ *bread* [20%, 60%].
  - Then find their lower-level "weaker" rules:
    - 1.5% *milk* ⇒ *wheat bread* [6%, 50%].



- Different min_support threshold across multi-levels lead to different algorithms:
  - adopting the same min_support across multi-levels
  - adopting reduced min_support at lower levels

Minimum Support for Multiple Levels

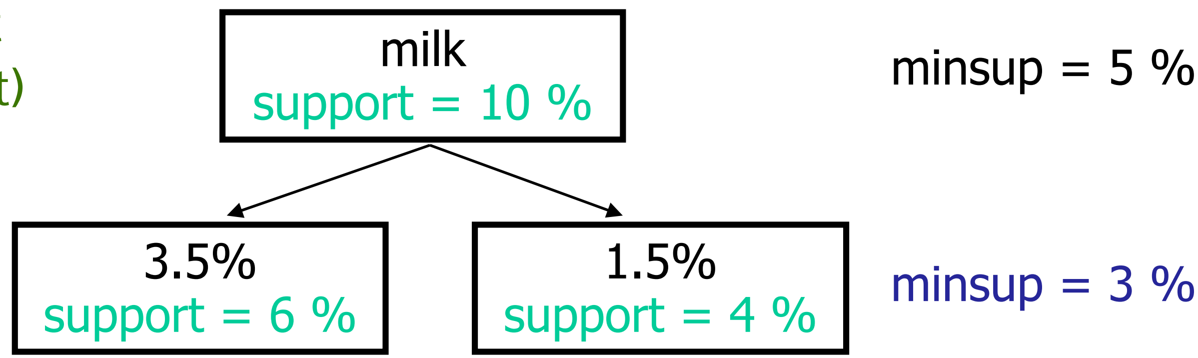
- Uniform Support



+ the search procedure is simplified (monotonicity)

+ the user is required to specify only one support threshold

- Reduced Support (Variable Support)

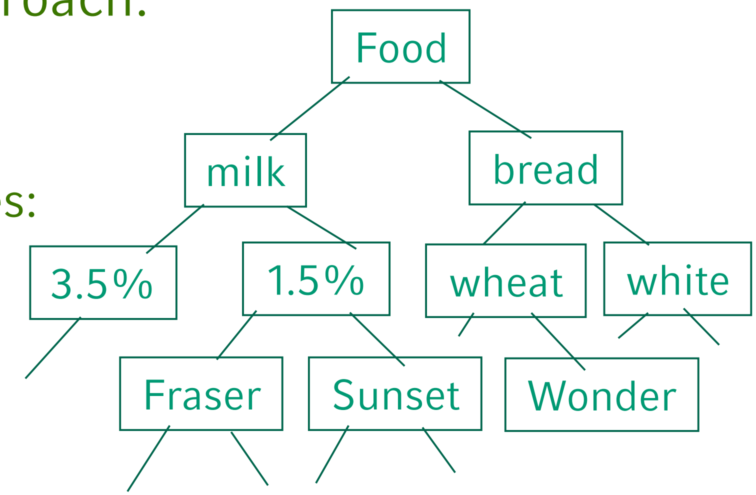


+ takes the lower frequency of items in lower levels into consideration

Multilevel Association Mining using Reduced Support

- A *top_down, progressive deepening* approach:
  - First find high-level strong rules:
    - *milk* ⇒ *bread* [20%, 60%].
  - Then find their lower-level "weaker" rules:
    - 1.5% *milk* ⇒ *wheat bread* [6%, 50%].

level-wise processing (breadth first)



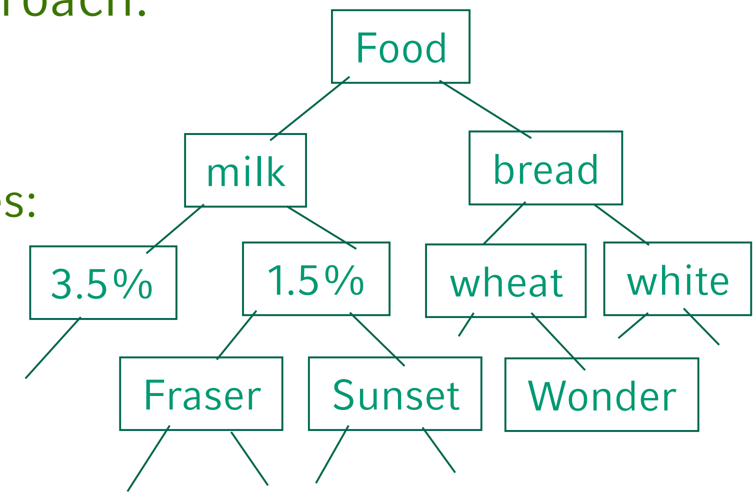
3 approaches using reduced Support:

- *Level-by-level independent method*:
  - Examine each node in the hierarchy, regardless of whether or not its parent node is found to be frequent
- *Level-cross-filtering by single item*:
  - Examine a node only if its parent node at the preceding level is frequent
- *Level-cross- filtering by k-itemset*:
  - Examine a k-itemset at a given level only if its parent k-itemset at the preceding level is frequent

Multilevel Associations: Variants

- A *top_down, progressive deepening* approach:
  - First find high-level strong rules:
    - *milk* ⇒ *bread* [20%, 60%].
  - Then find their lower-level "weaker" rules:
    - 1.5% *milk* ⇒ *wheat bread* [6%, 50%].

level-wise processing (breadth first)



- Variations at mining multiple-level association rules.
  - Level-crossed association rules:
    - 1.5 % *milk* ⇒ *Wonder wheat bread*
  - Association rules with multiple, alternative hierarchies:
    - 1.5 % *milk* ⇒ *Wonder bread*

Multi-level Association: Redundancy Filtering

- Some rules may be redundant due to “ancestor” relationships between items.
- Example
  - R_1: milk $\Rightarrow$ wheat bread [support = 8%, confidence = 70%]
  - R_2: 1.5% milk $\Rightarrow$ wheat bread [support = 2%, confidence = 72%]
- We say that rule 1 is an ancestor of rule 2.
- *Redundancy:*
  A rule is redundant if its support is close to the “expected” value, based on the rule’s ancestor.

Interestingness of Hierarchical Association Rules: Notions

Let $X, X', Y, Y' \subseteq I$ be itemsets.

- An itemset X' is an ancestor of X iff there exist ancestors $x_1', \dots, x_k'$ of $x_1, \dots, x_k \in X$ and $x_{k+1}, \dots, x_n$ with $n = |X|$ such that
$$X' = \{x_1', \dots, x_k', x_{k+1}, \dots, x_n\}.$$
- Let X' and Y' be ancestors of X and Y. Then we call the rules $X' \Rightarrow Y'$, $X \Rightarrow Y'$, and $X' \Rightarrow Y$ *ancestors* of the rule X ⇒ Y .
- The rule X´ ⇒ Y´ is a *direct ancestor* of rule X ⇒ Y in a set of rules if:
  - Rule X´ ⇒ Y' is an ancestor of rule X ⇒ Y, and
  - There is no rule X" ⇒ Y" such that X" ⇒ Y" is an ancestor of X ⇒ Y and X´ ⇒ Y´ is an ancestor of X" ⇒ Y"
- A hierarchical association rule X ⇒ Y is called *R-interesting* if:
  - There are no direct ancestors of X ⇒ Y or
  - The actual support is larger than *R* times the expected support or
  - The actual confidence is larger than *R* times the expected confidence

Expected Support and Expected Confidence

- How to compute the expected support?
  Given the rule for X ⇒ Y and its ancestor rule X´ ⇒ Y´ the expected support of X ⇒ Y is defined as:
  $$E_{Z'}[\mathrm{P}(Z)] = \frac{\mathrm{P}(z_1)}{\mathrm{P}(z_1')} \times \cdots \times \frac{\mathrm{P}(z_j)}{\mathrm{P}(z_j')} \times \mathrm{P}(Z')$$
  where $Z = X \cup Y = \{z_1, \ldots, z_n\}$, $Z' = X' \cup Y' = \{z_1', \ldots, z_j', z_{j+1}, \ldots, z_n\}$ and each $z_i' \in Z'$ is an ancestor of $z_i \in Z$

[SA'95] R. Srikant, R. Agrawal: *Mining Generalized Association Rules*. In VLDB, 1995.

Expected Support and Expected Confidence

- How to compute the expected confidence?
  Given the rule for X ⇒ Y and its ancestor rule X´ ⇒ Y´, then the expected confidence of X ⇒ Y is defined as:
  $$E_{X' \Rightarrow Y'}[\mathrm{P}(Y|X)] = \frac{\mathrm{P}(y_1)}{\mathrm{P}(y_1')} \times \cdots \times \frac{\mathrm{P}(y_j)}{\mathrm{P}(y_j')} \times \mathrm{P}(Y'|X')$$
  where $Y = \{y_1, \ldots, y_n\}$ and $Y' = \{y_1', \ldots, y_j', y_{j+1}, \ldots, y_n\}$ and each $y_i' \in Y'$ is an ancestor of $y_i \in Y$

[SA'95] R. Srikant, R. Agrawal: *Mining Generalized Association Rules*. In VLDB, 1995.

Interestingness of Hierarchical Association Rules:Example

- Example
  - Let $R = 1.6$

Item	Support
clothes	20
outerwear	10
jackets	4

No	rule	support	R-interesting?
1	clothes $\Rightarrow$ shoes	10	yes: no ancestors
2	outerwear $\Rightarrow$ shoes	9	yes: Support $>$ R *exp. support (wrt. rule 1) = $(1.6 \cdot (\frac{10}{20} \cdot 10)) = 8$
3	jackets $\Rightarrow$ shoes	4	Not wrt. support: Support > R * exp. support (wrt. rule 1) = 3.2 Support < R * exp. support (wrt. rule 2) = 5.75 → still need to check the confidence!

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Multi-Dimensional Association: Concepts

- Single-dimensional rules:
  - buys milk $\Rightarrow$ buys bread

- Multi-dimensional rules: $\geq$ 2 dimensions
  - Inter-dimension association rules (*no repeated dimensions*)
    - age between 19-25 $\wedge$ status is student $\Rightarrow$ buys coke
  - hybrid-dimension association rules (*repeated dimensions*)
    - age between 19-25 $\wedge$ buys popcorn $\Rightarrow$ buys coke

Techniques for Mining Multi-Dimensional Associations

- Search for frequent *k*-predicate set:
  - Example: {age, occupation, buys} is a 3-predicate set.
  - Techniques can be categorized by how age is treated.

1. Using static discretization of quantitative attributes
   - Quantitative attributes are statically discretized by using predefined concept hierarchies.
2. Quantitative association rules
   - Quantitative attributes are dynamically discretized into "bins"based on the distribution of the data.
3. Distance-based association rules
   - This is a dynamic discretization process that considers the distance between data points.

Quantitative Association Rules

- Up to now: associations of *boolean* attributes only
- Now: *numerical* attributes, too
- Example:
  - Original database

ID	age	marital status	# cars
1	23	single	0
2	38	married	2

  - Boolean database

ID	age: 20..29	age: 30..39	m-status: single	m-status: married	. . .
1	1	0	1	0	. . .
2	0	1	0	1	. . .

Quantitative Association Rules: Ideas

- Static discretization
  - Discretization of all attributes *before* mining the association rules
  - E.g. by using a generalization hierarchy for each attribute
  - Substitute numerical attribute values by ranges or intervals

- Dynamic discretization
  - Discretization of the attributes *during* association rule mining
  - Goal (e.g.): maximization of confidence
  - Unification of neighboring association rules to a generalized rule

Partitioning of Numerical Attributes

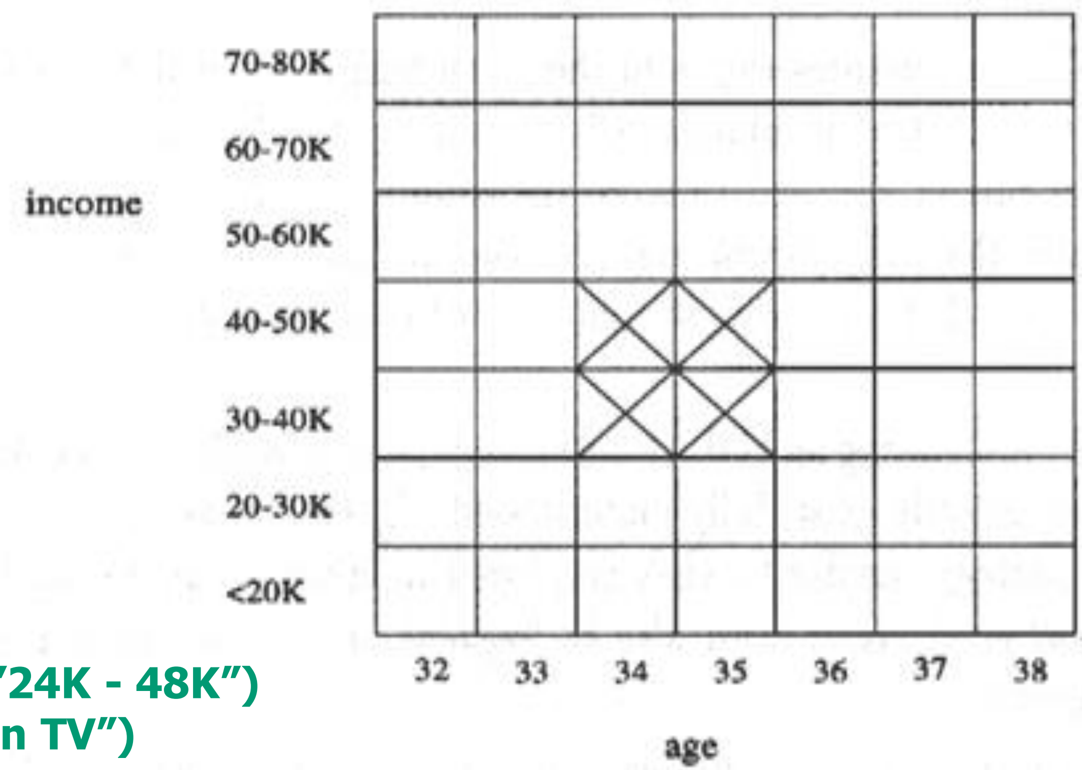
- Problem: Minimum support
  - Too many intervals $\rightarrow$ too small support for each individual interval
  - Too few intervals $\rightarrow$ too small confidence of the rules

- Solution
  - First, partition the domain into many intervals
  - Afterwards, create new intervals by merging adjacent interval

- Numeric attributes are *dynamically* discretized such that the confidence or compactness of the rules mined is maximized.

Quantitative Association Rules

- 2-D quantitative association rules: $A_{quan1} \wedge A_{quan2} \Rightarrow A_{cat}$
- Cluster "adjacent" association rules to form general rules using a 2-D grid.
- Example:



age(X,"30-34") $\wedge$ income(X,"24K - 48K")
$\Rightarrow$ buys(X,"high resolution TV")

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Chapter 3: Summary

- Mining frequent itemsets
  - Apriori algorithm, hash trees, FP-tree
- Simple association rules
  - support, confidence, rule generation, interestingness measures (correlation), …
- Further topics
  - Hierarchical association rules: algorithms (top-down progressive deepening), multilevel support thresholds, redundancy and R-interestingness
  - Quantitative association rules: partitioning numerical attributes, adaptation of apriori algorithm, interestingness
- Extensions: multi-dimensional association rule mining