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Knowledge Discovery in Databases

SS 2012

Übungsblatt 8: Cluster Analysis: EM and OPTICS

Aufgabe 8-1 *EM-Algorithm*

Given a data set with 100 points consisting of three Gaussian clusters A, B and C and the point p.

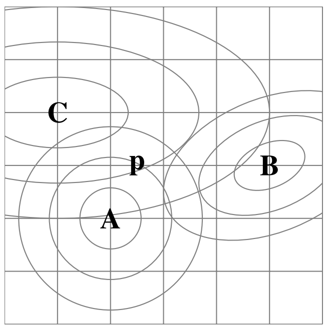
The cluster A contains 30% of all objects and is represented using the mean of all his points $\mu_A = (2, 2)$ and the covariance matrix $\Sigma_A = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$.

The cluster B contains 20% of all objects and is represented using the mean of all his points $\mu_B = (5, 3)$ and the covariance matrix $\Sigma_B = \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix}$.

The cluster C contains 50% of all objects and is represented using the mean of all his points $\mu_C = (1, 4)$ and the covariance matrix $\Sigma_C = \begin{pmatrix} 16 & 0 \\ 0 & 4 \end{pmatrix}$.

The point p is given by the coordinates $(2.5, 3.0)$.
Compute the three probabilities of p belonging to the clusters A, B and C.

The following sketch is not exact, and only gives a rough idea of the cluster locations:



Aufgabe 8-2 *Multivariate Density and Mahalanobis Distance*

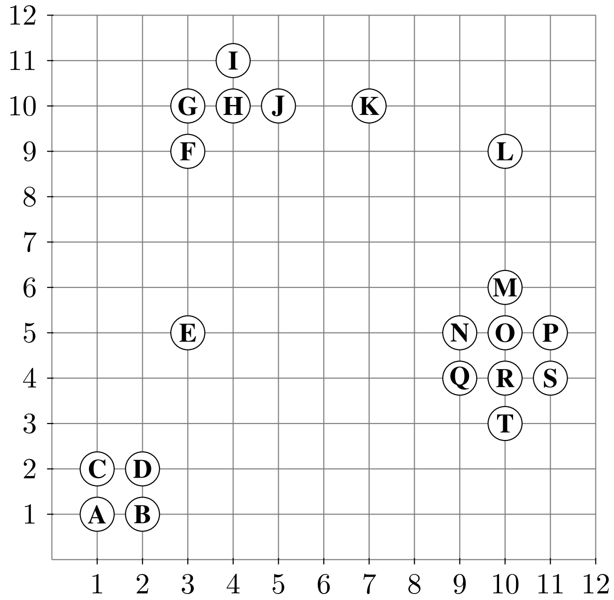
The density of the multivariate normal distribution (with Σ, μ) is computed using the formula

$$prob(p, \mu, \Sigma) := \frac{1}{\sqrt{(2\pi)^d |\Sigma|}} \cdot e^{-\frac{1}{2}\left((p-\mu)^T \Sigma^{-1} (p-\mu)\right)}$$

Find and discuss the relationship of the formula to the Mahalanobis distance (using Σ) of p to μ.

$$d_{\mathit{Mahalanobis}}(x, y, \Sigma) := \sqrt{(x-y)^T \Sigma^{-1} (x-y)}$$

Aufgabe 8-3 *OPTICS*



As distance function, use Manhattan distance $L_1(a,b) := |a_1 - b_1| + |a_2 - b_2|$.
Construct an OPTICS reachability plot (see pseudo-code below) for each of the following parameter settings:

- $\varepsilon = 5$ and *minPts* $= 2$
- $\varepsilon = 5$ and *minPts* $= 4$
- $\varepsilon = 2$ and *minPts* $= 4$
- $\varepsilon = \infty$ and *minPts* $= 4$

Pseudocode OPTICS

```
seedlist = ∅    // implemented as a heap
for i = 0 to n-1 do
    if(seedlist = ∅ )  then seedlist = {(random_not_handled_point, ∞)}
    (x, x.reach) = get_and_remove_point_with_min_reach(seedlist)
    x.pos = i
    x.handled = TRUE
    neighbors = rangeQuery(x, ε)
    x.core = nnDist(x, neighbors, MinPts)
    if(x.core < ∞)
        for each y ∈ neighbors with not(y.handled)
            if( y ∉ seedlist)  seedlist = seedlist ∪ {(y, reach-dist(y,x))}
            else
                curr_reach = lookup(seedlist, y)
                update(y, min(curr_reach, reach-dist(y,x)))
        endfor
endfor
```