Data Mining Tutorial

Session 6: Outlier Detection

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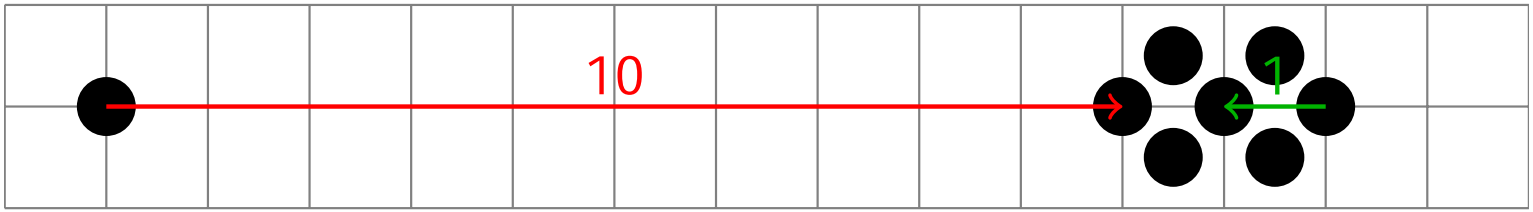
2012-07-05 — KDD class tutorial

Distance based outlier models

Distance based outliers:
"Outlier are further away from the data"

- Distance to nearest neighbor



Distance based outlier models

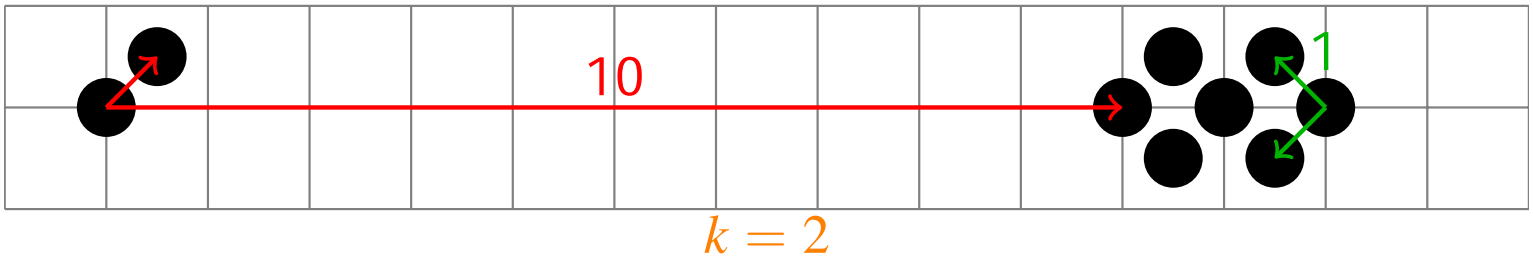
Distance based outliers:
"Outlier are further away from the data"

- Distance to nearest neighbor
  $\Rightarrow$ misses paired outliers

Distance based outlier models

Distance based outliers:
"Outlier are further away from the data"

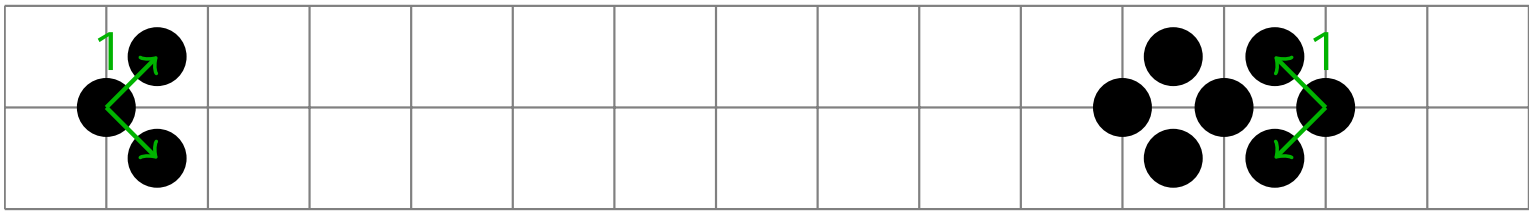
- Distance to k nearest neighbor



Distance based outlier models

Distance based outliers:
"Outlier are further away from the data"

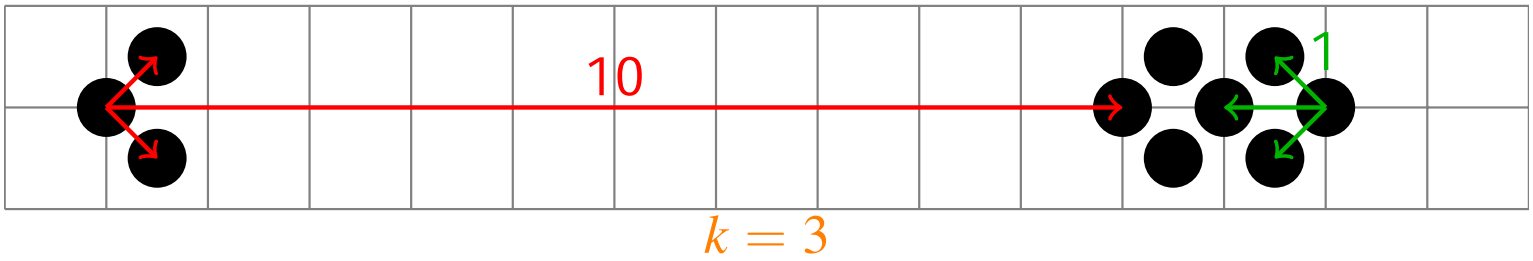
- Distance to k nearest neighbor



Distance based outlier models

Distance based outliers:
"Outlier are further away from the data"

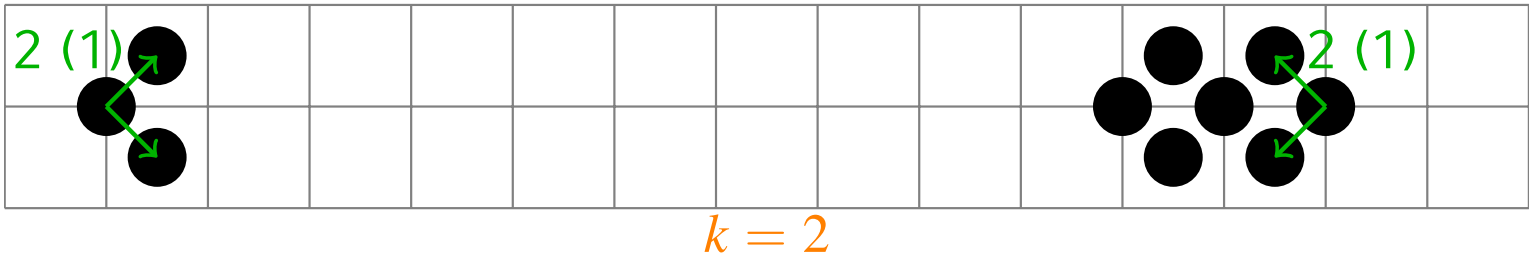
- Distance to k nearest neighbor
  $\Rightarrow$ micro clusters ($|C| < k + 1$) become outliers



Distance based outlier models

Distance based outliers:
"Outlier are further away from the data"

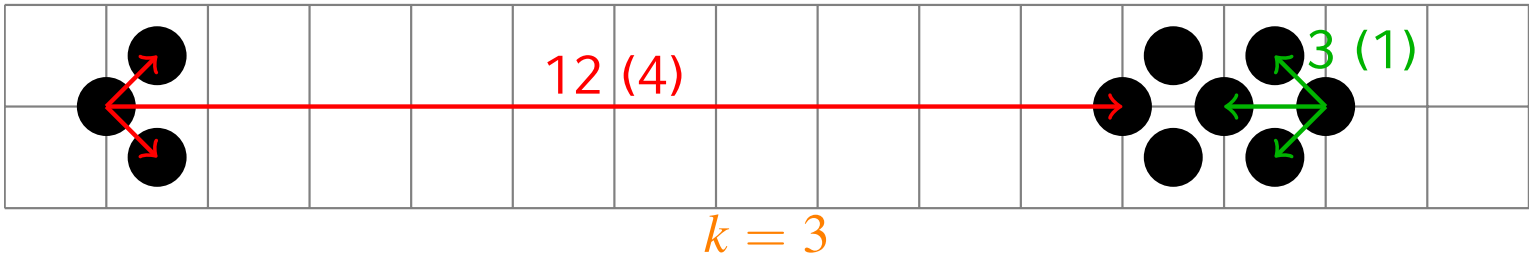
- Sum of distances to the first k nearest neighbors
  More robust with respect to k and micro clusters



Distance based outlier models

Distance based outliers:
"Outlier are further away from the data"

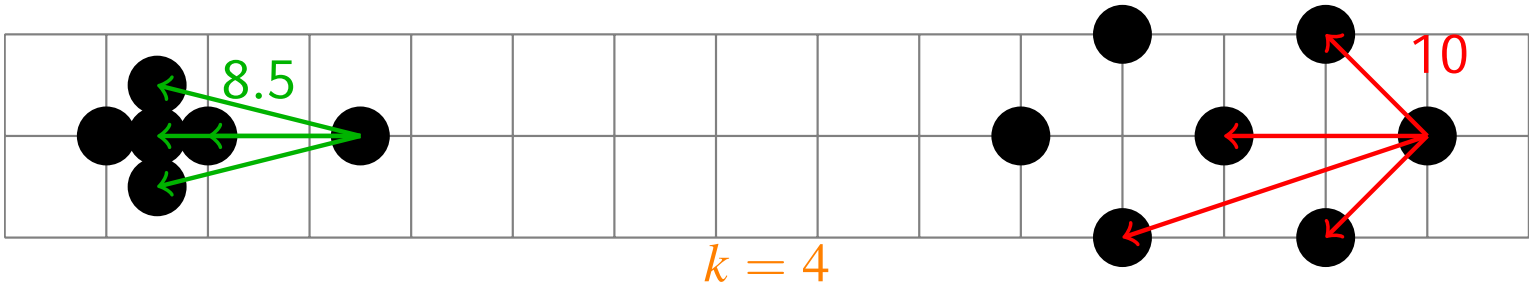
- Sum of distances to the first k nearest neighbors



Distance based outlier models

Distance based outliers:
"Outlier are further away from the data"

- Sum of distances to the first k nearest neighbors
  $\Rightarrow$ Cannot handle different densities



Local Outlier Factor

Local outlier factor:

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Idea: Outliers are less dense than their neighbors

Local Outlier Factor

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Density estimation using ideas from OPTICS/DBSCAN:

"When would the object p be in the *minPts* $= k$ core *of* o?"

Local Outlier Factor

Local outlier factor:

Idea: Outliers are less dense than their neighbors

Density estimation using ideas from OPTICS/DBSCAN:

"When would the object p be in the *minPts* $= k$ core *of* o?"

$$\textit{reach-dist}_k(p, o) = \max\{k\text{-}\textit{distance}(o), \textit{dist}(o, p)\}$$

Local Outlier Factor

Local outlier factor:

Idea: Outliers are less dense than their neighbors

Density estimation using ideas from OPTICS/DBSCAN:

"When would the object p be in the *minPts* $= k$ core *of* o?"

$$\textit{reach-dist}_k(p,o) = \max\{\underbrace{k\textit{-distance}(o)}_{k\text{NN outlier!}}, \textit{dist}(o,p)\}$$

Local Outlier Factor

Local outlier factor:
Idea: Outliers are less dense than their neighbors
Density estimation using ideas from OPTICS/DBSCAN:
"When would the object p be in the $minPts = k$ core *of* o?"

$$\textit{reach-dist}_k(p \leftarrow o) = \max\{k\text{-}\textit{distance}(o), \textit{dist}(o,p)\}$$

Be careful: this is not symmetric.
But the "core size of the *other* object o"
I prefer the notion of

$$\textit{reach-dist}_k(p \leftarrow o)$$

to emphasize the direction.

Local Outlier Factor

Local outlier factor:

Idea: Outliers are less dense than their neighbors

Density estimation using ideas from OPTICS/DBSCAN:

"When would the object p be in the *minPts* $= k$ core *of* o?"

$$\textit{reach-dist}_k(p \leftarrow o) = \max\{k\text{-}\textit{distance}(o), \textit{dist}(o,p)\}$$

Local reachability density

$$\textit{lrd}_k(p) = 1 \Big/ \left(\frac{\sum_{o \in \mathcal{N}} \textit{reach-dist}_k(p \leftarrow o)}{|\mathcal{N}|} \right)$$

Local Outlier Factor

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$$\textit{reach-dist}_k(p \leftarrow o) = \max\{k\text{-}\textit{distance}(o), \textit{dist}(o,p)\}$$

Local reachability density

$$1/\textit{lrd}_k(p) = \underbrace{\frac{1}{|\mathcal{N}|}\sum_{o\in\mathcal{N}}}_{\text{average}} \underbrace{\textit{reach-dist}_k(p \leftarrow o)}_{\text{reachability } \textit{from} \text{ all neighbors}}$$

Note: I prefer computing it as $1/\textit{lrd}$, avoids division by 0.
Plus: it will look like $(a+b+c+d)/\textit{count}$ – typical average!

Local Outlier Factor

$$LOF(p) = \frac{\sum_{o \in \mathcal{N}} \frac{lrd_k(o)}{lrd_k(p)}}{|\mathcal{N}|}$$

Local Outlier Factor

$$LOF(p) = \frac{1}{|\mathcal{N}|} \sum_{o \in \mathcal{N}} \frac{lrd_k(o)}{lrd_k(p)}$$

Local Outlier Factor

$$LOF(p) = \underbrace{\frac{1}{|\mathcal{N}|}\sum_{o\in\mathcal{N}}}_{\text{average}} \underbrace{\frac{lrd_k(o)}{lrd_k(p)}}_{\text{relative density}}$$

Local Outlier Factor

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- Same density $\Leftrightarrow$ relative density $= 1$

Local Outlier Factor

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- Less dense $\Leftrightarrow$ relative density > 1

Local Outlier Factor

$$LOF(p) = \underbrace{\frac{1}{|\mathcal{N}|}\sum_{o\in\mathcal{N}}}_{\text{average}} \underbrace{\frac{lrd_k(o)}{lrd_k(p)}}_{\text{relative density}}$$

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- Less dense $\Leftrightarrow$ relative density > 1
- $LOF(p) \gg 1$ for outliers!

Local Outlier Factor

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- $LOF(p) \gg 1$ for outliers!

Note: Density in the OPTICS/DBSCAN sense!
The "reachability distance" is often overlooked.

Local Outlier Factor

$$LOF(p) = \underbrace{\frac{1}{|\mathcal{N}|}\sum_{o\in\mathcal{N}}}_{\text{average}} \underbrace{\frac{lrd_k(o)}{lrd_k(p)}}_{\text{relative density}}$$

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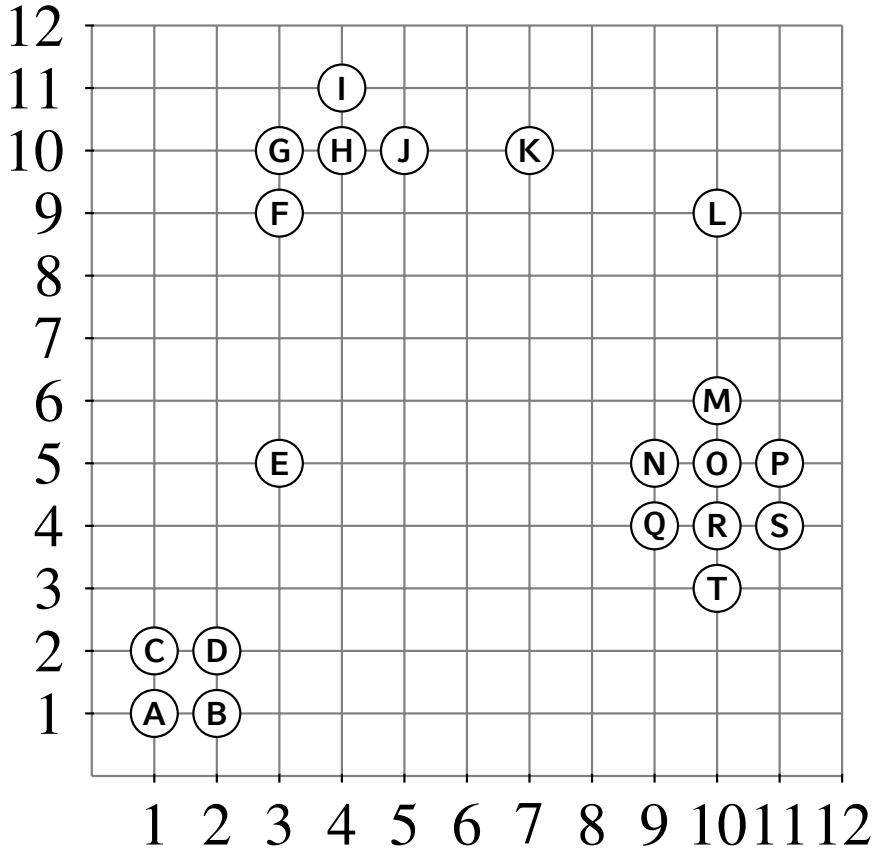
- Less dense $\Leftrightarrow$ relative density > 1
- $LOF(p) \gg 1$ for outliers!

Note: Density in the OPTICS/DBSCAN sense!
The "reachability distance" is often overlooked.
Division by 0 only occurs when k objects have distance 0!

LOF and kNN



	2NN	2d.	4NN	4d.
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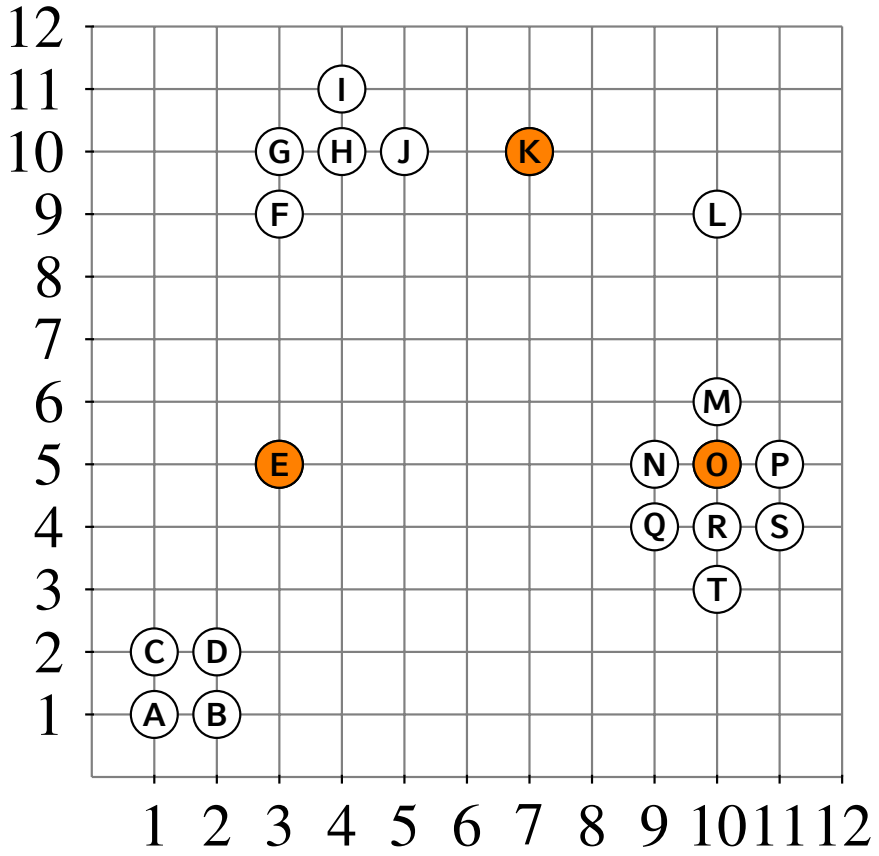
LOF and kNN



	2NN	2d.	4NN	4d.
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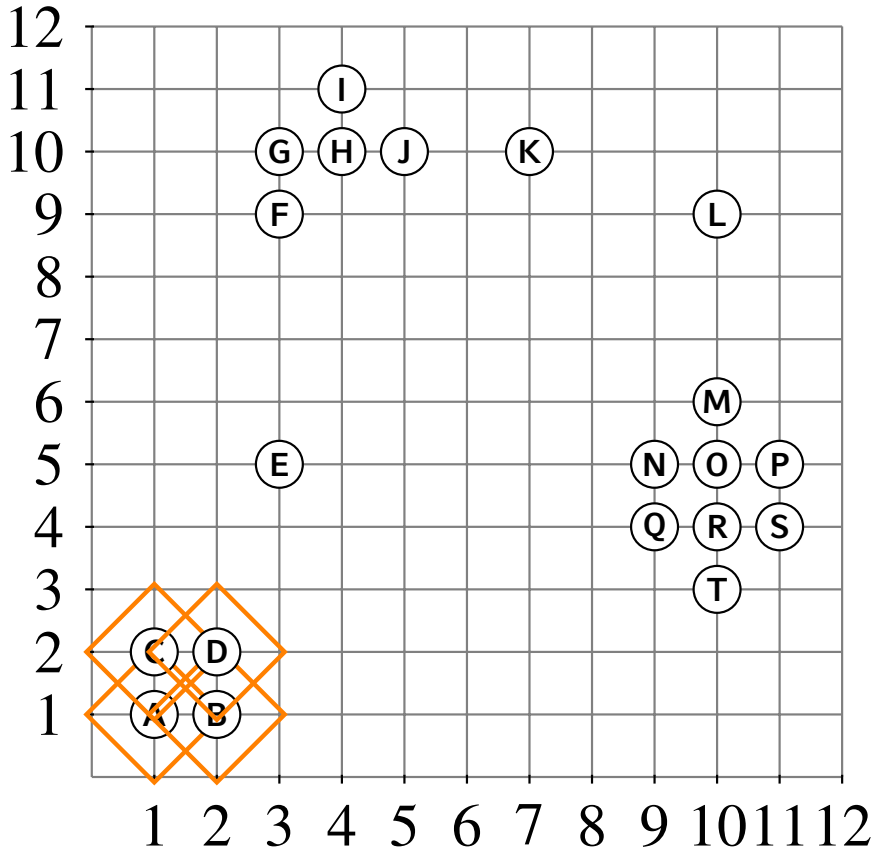
LOF and kNN



	2NN	2d.	4NN	4d.
A	B C	1		
B	A D	1		
C	A D	1		
D	B C	1		
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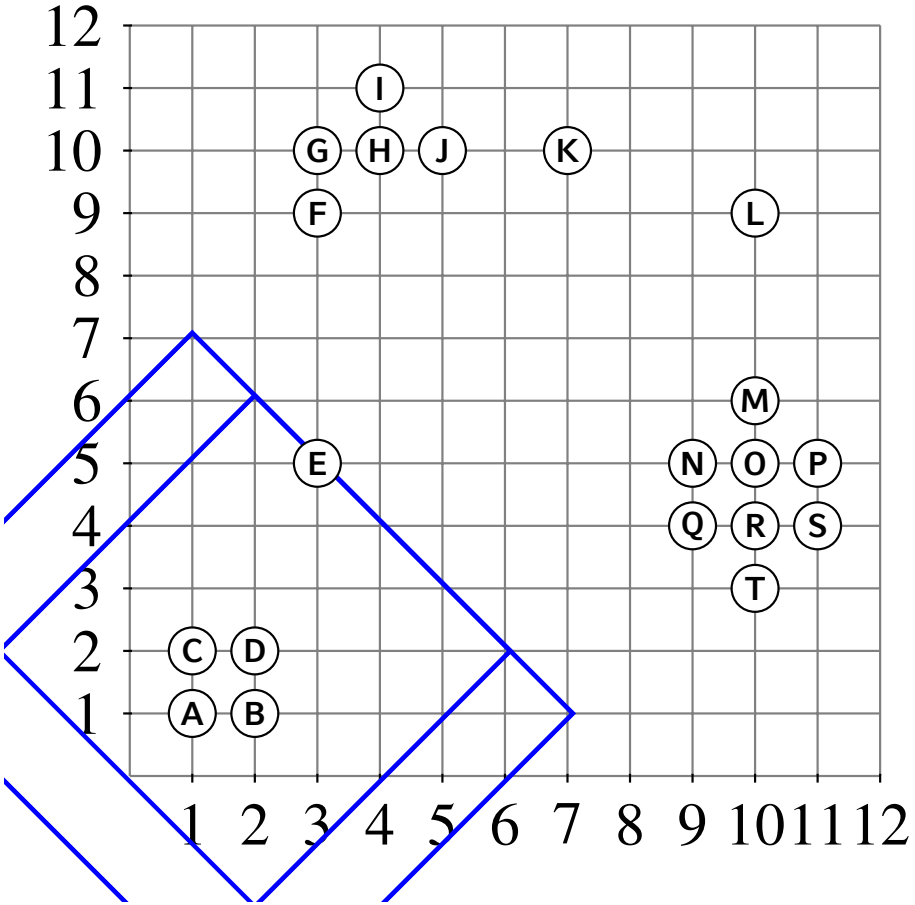
LOF and kNN



	2NN	2d.	4NN	4d.
A	B C	1	B C D E	6
B	A D	1	A C D E	5
C	A D	1	A B D E	5
D	B C	1	A B C E	4
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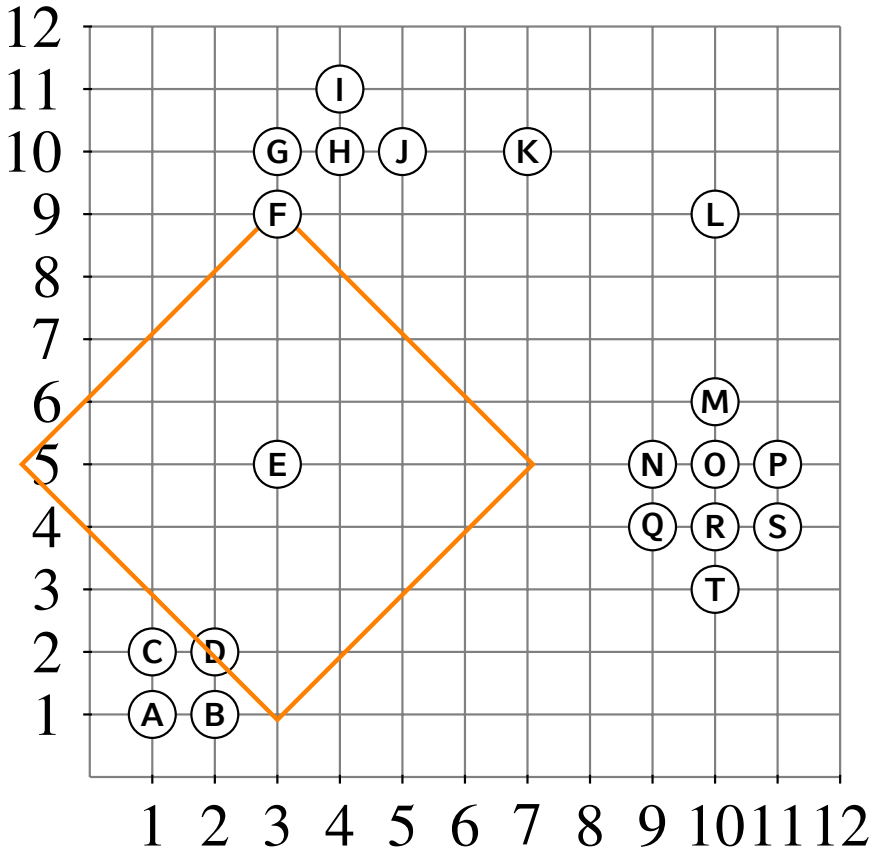
LOF and kNN



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A	B C	1	B C D E	6
B	A D	1	A C D E	5
C	A D	1	A B D E	5
D	B C	1	A B C E	4
E	D F	4		
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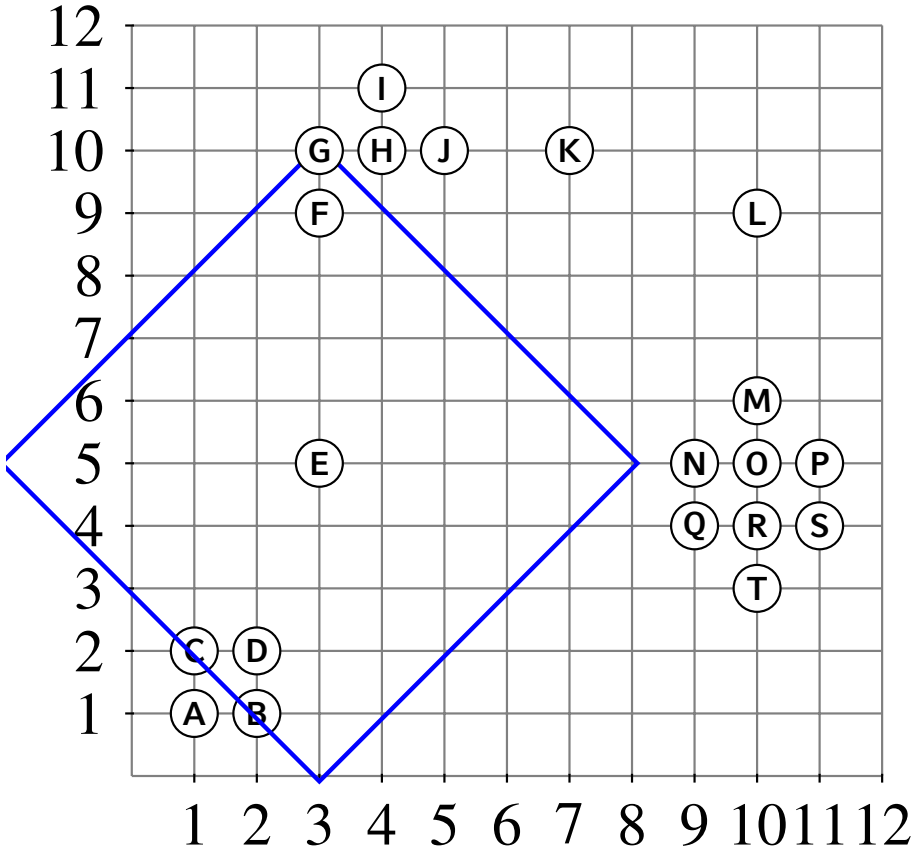

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D	B C	1	A B C E	4
E	D F	4	B C D F G	5
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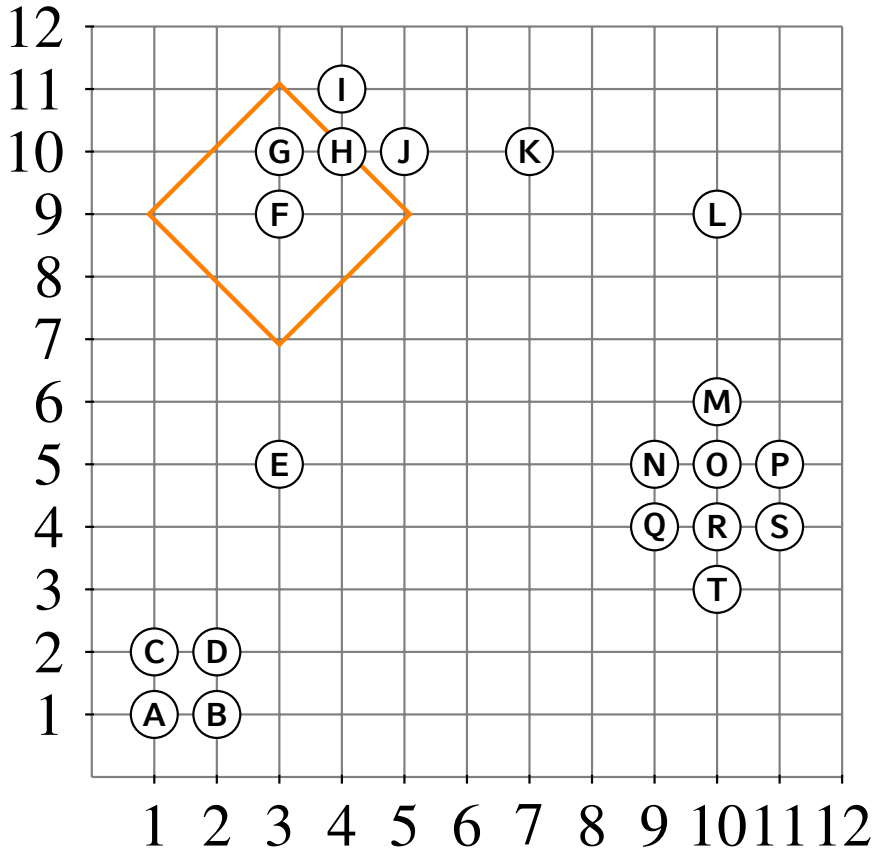

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B	A D	1	A C D E	5
C	A D	1	A B D E	5
D	B C	1	A B C E	4
E	D F	4	B C D F G	5
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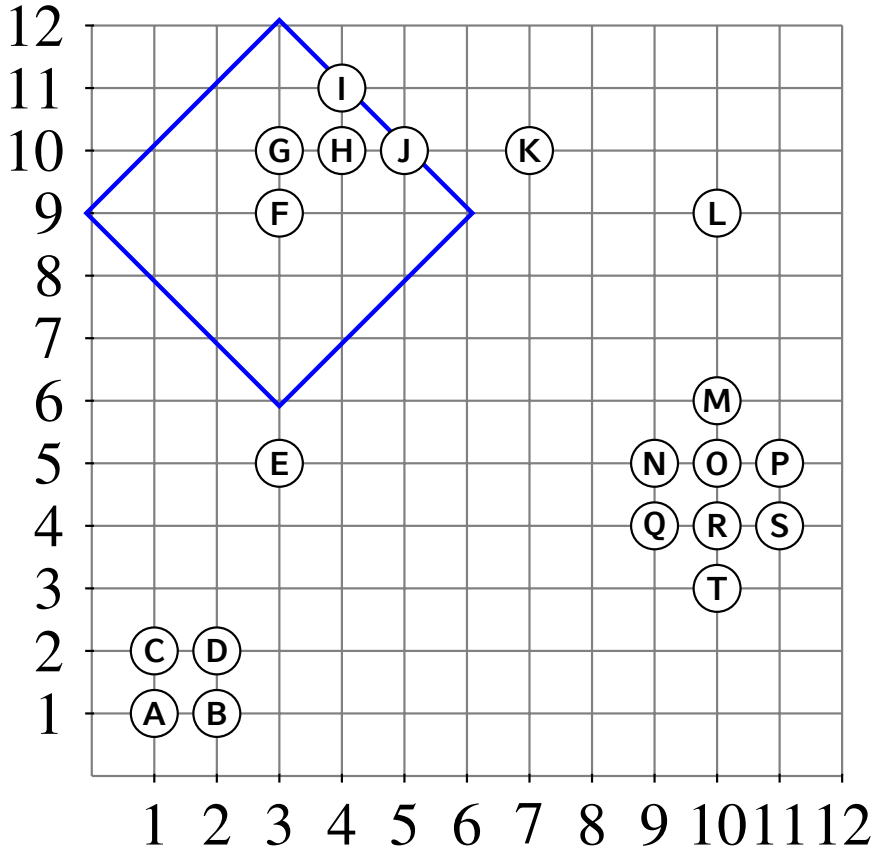
LOF and kNN



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B	A D	1	A C D E	5
C	A D	1	A B D E	5
D	B C	1	A B C E	4
E	D F	4	B C D F G	5
F	G H	2	G H I J	3
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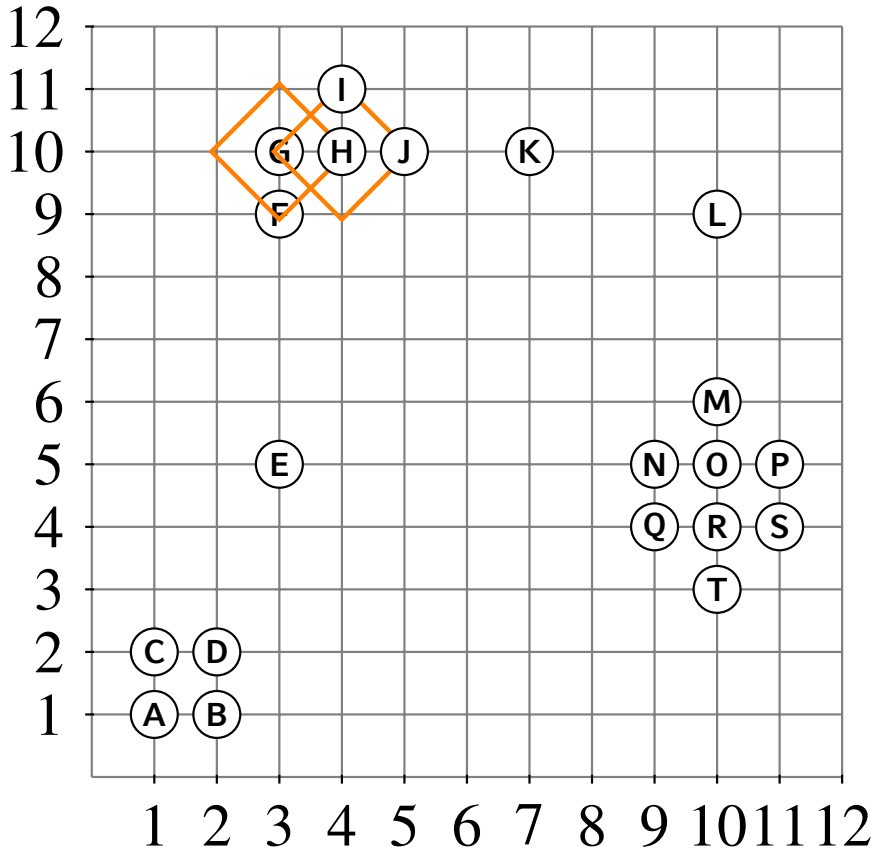
LOF and kNN



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B	A D	1	A C D E	5
C	A D	1	A B D E	5
D	B C	1	A B C E	4
E	D F	4	B C D F G	5
F	G H	2	G H I J	3
G	F H	1		
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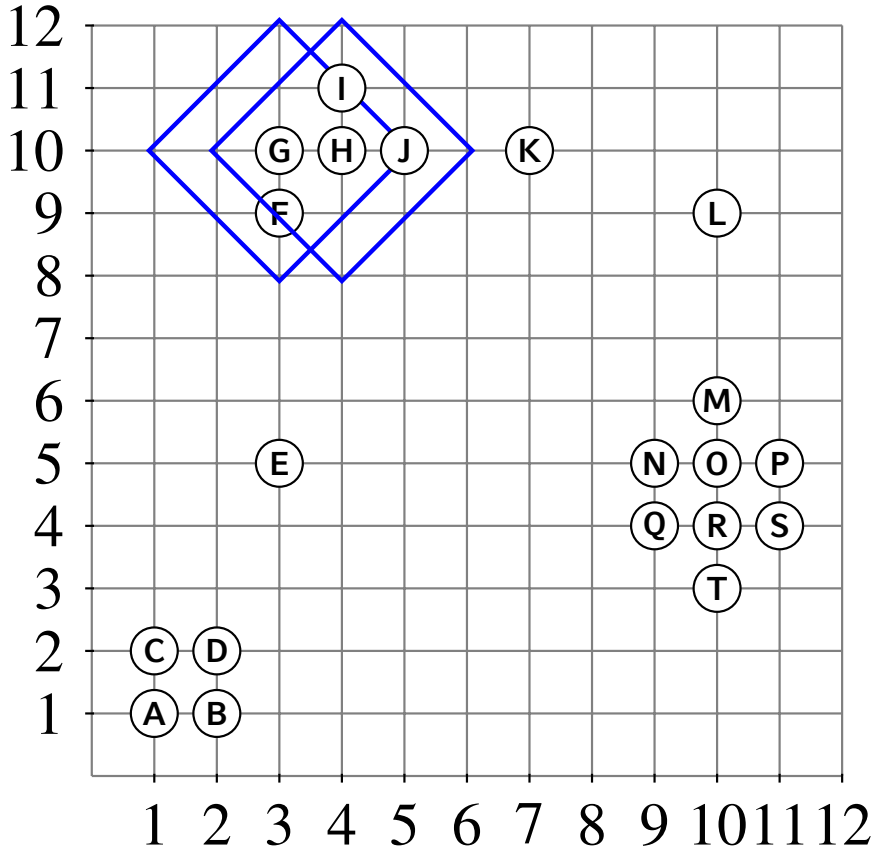

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B	A D	1	A C D E	5
C	A D	1	A B D E	5
D	B C	1	A B C E	4
E	D F	4	B C D F G	5
F	G H	2	G H I J	3
G	F H	1	F H I J	2
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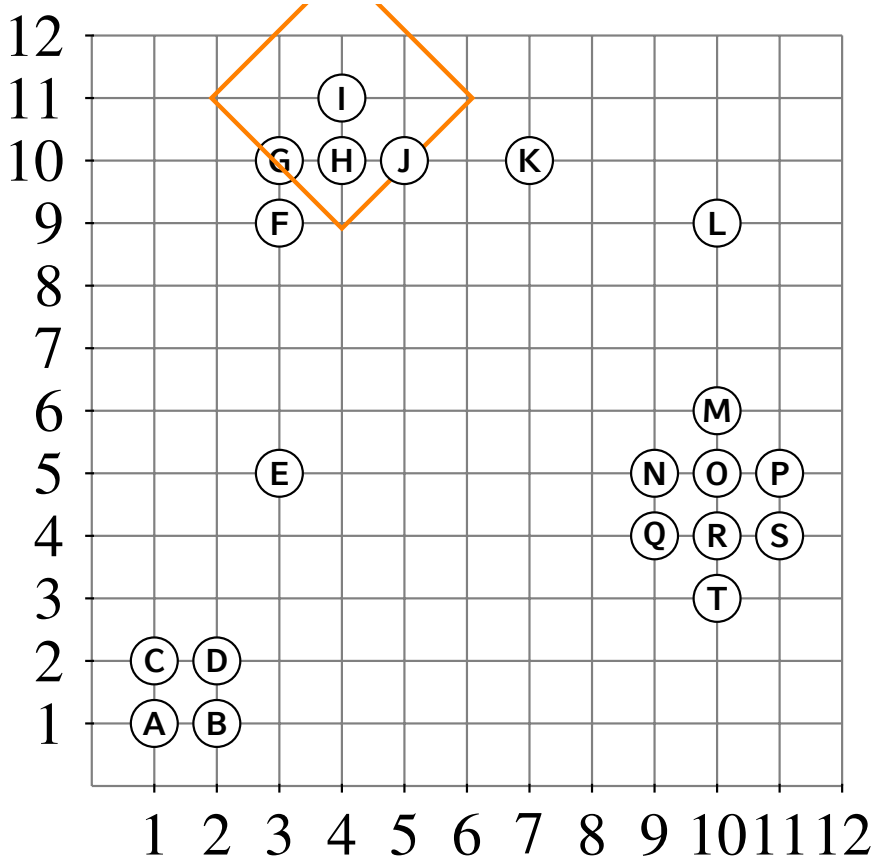
LOF and kNN



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B	A D	1	A C D E	5
C	A D	1	A B D E	5
D	B C	1	A B C E	4
E	D F	4	B C D F G	5
F	G H	2	G H I J	3
G	F H	1	F H I J	2
H	G I J	1	F G I J	2
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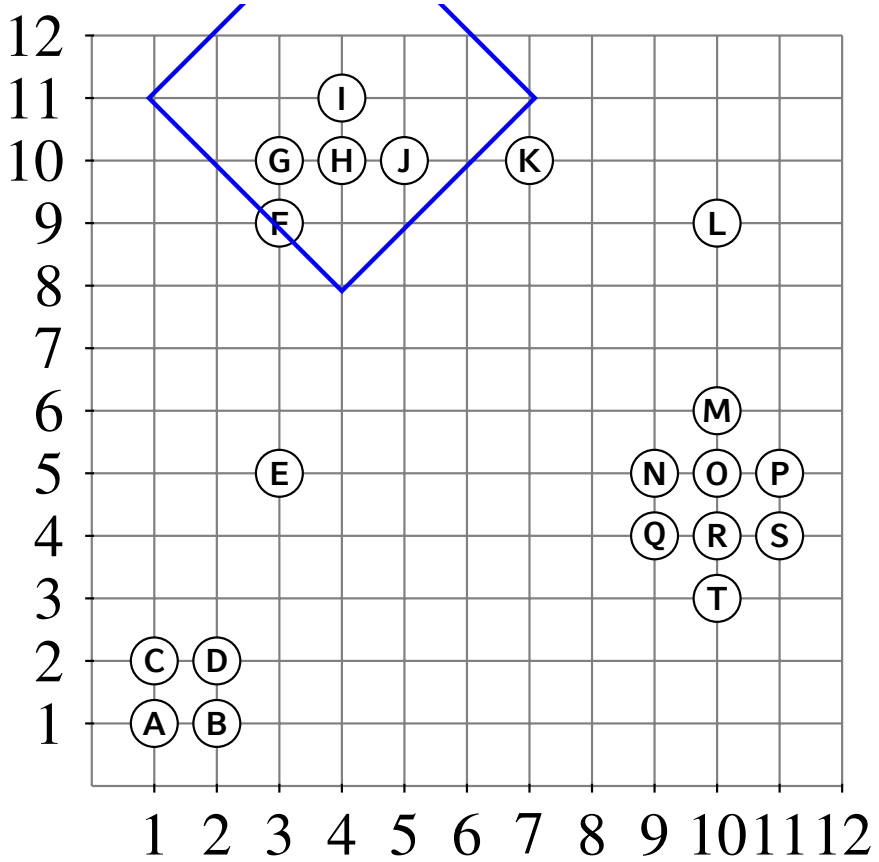

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D	B C	1	A B C E	4
E	D F	4	B C D F G	5
F	G H	2	G H I J	3
G	F H	1	F H I J	2
H	G I J	1	F G I J	2
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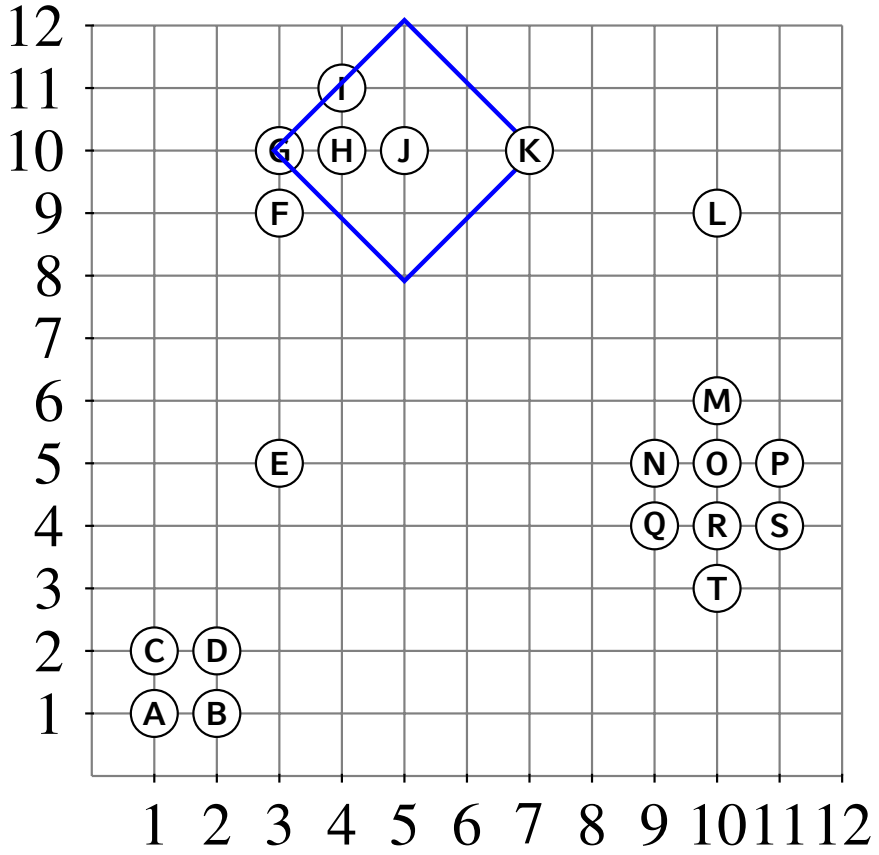
LOF and kNN



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C	A D	1	A B D E	5
D	B C	1	A B C E	4
E	D F	4	B C D F G	5
F	G H	2	G H I J	3
G	F H	1	F H I J	2
H	G I J	1	F G I J	2
I	G H J	2	F G H J	3
J	G H I K	2	G H I K	2
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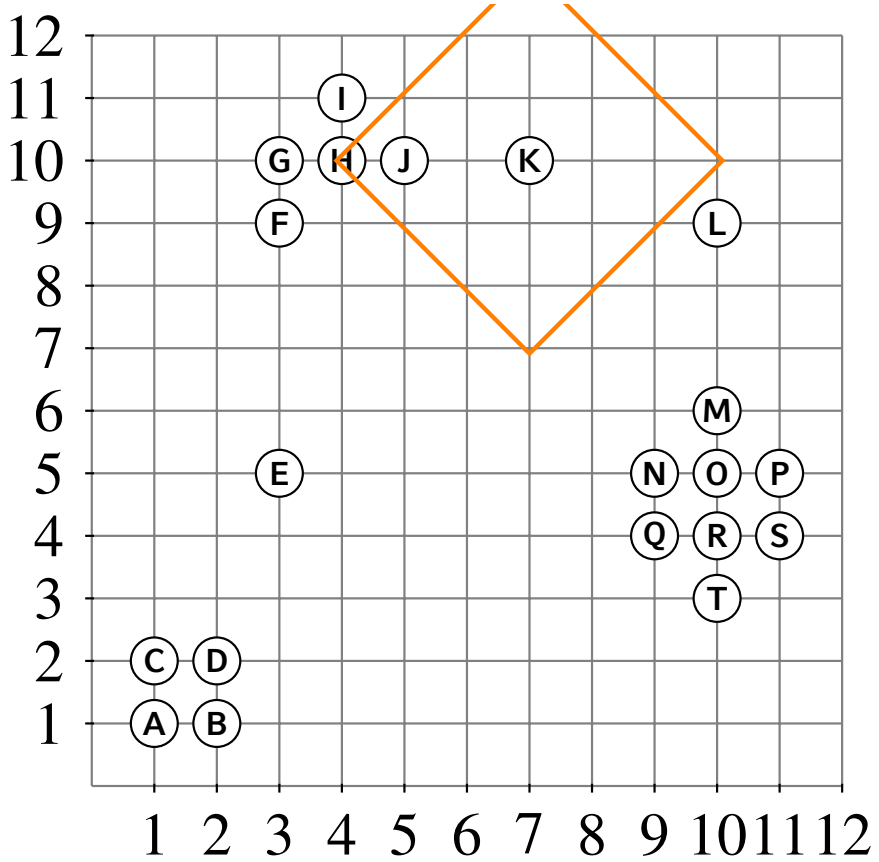

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G	F H	1	F H I J	2
H	G I J	1	F G I J	2
I	G H J	2	F G H J	3
J	G H I K	2	G H I K	2
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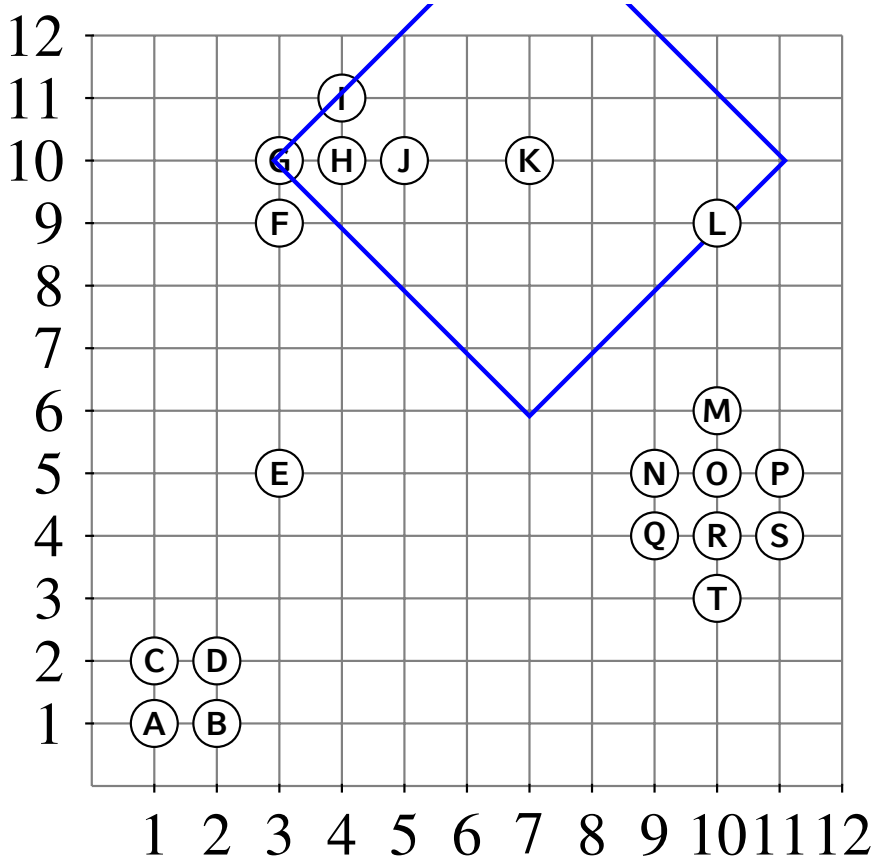
LOF and kNN



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E	D F	4	B C D F G	5
F	G H	2	G H I J	3
G	F H	1	F H I J	2
H	G I J	1	F G I J	2
I	G H J	2	F G H J	3
J	G H I K	2	G H I K	2
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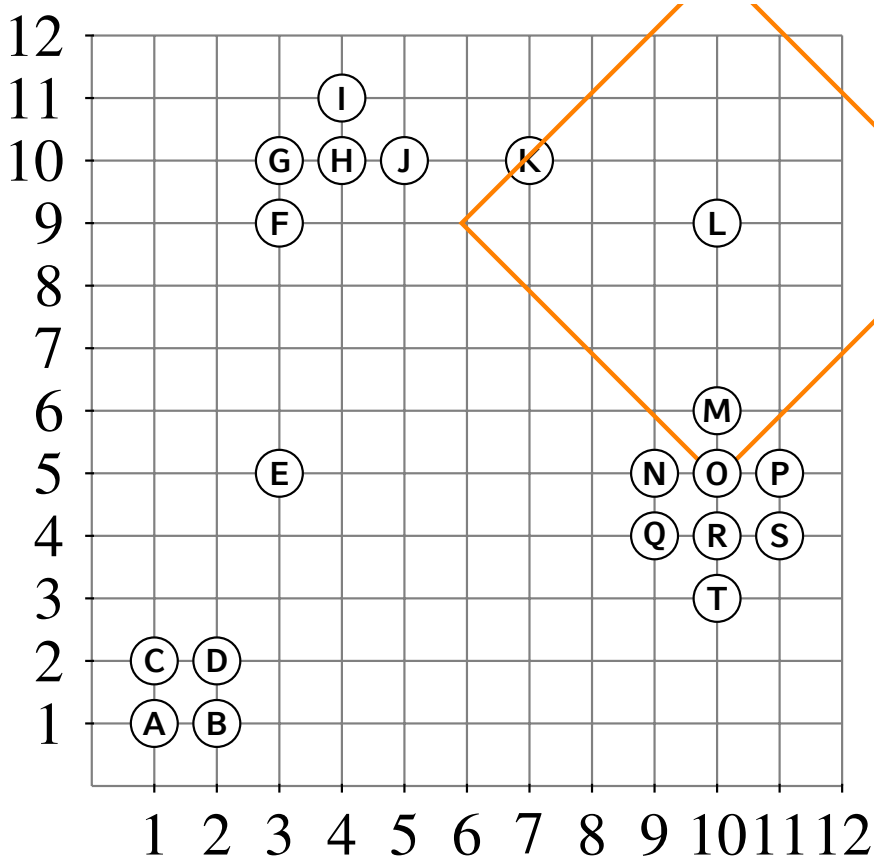
LOF and kNN



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D	B C	1	A B C E	4
E	D F	4	B C D F G	5
F	G H	2	G H I J	3
G	F H	1	F H I J	2
H	G I J	1	F G I J	2
I	G H J	2	F G H J	3
J	G H I K	2	G H I K	2
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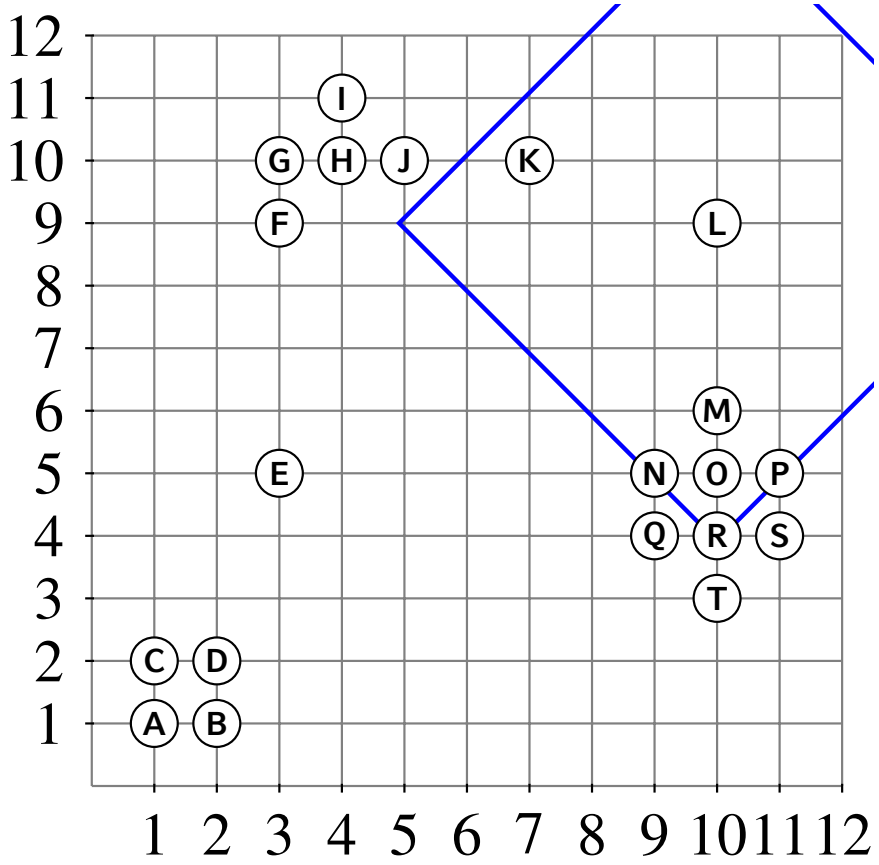
LOF and kNN



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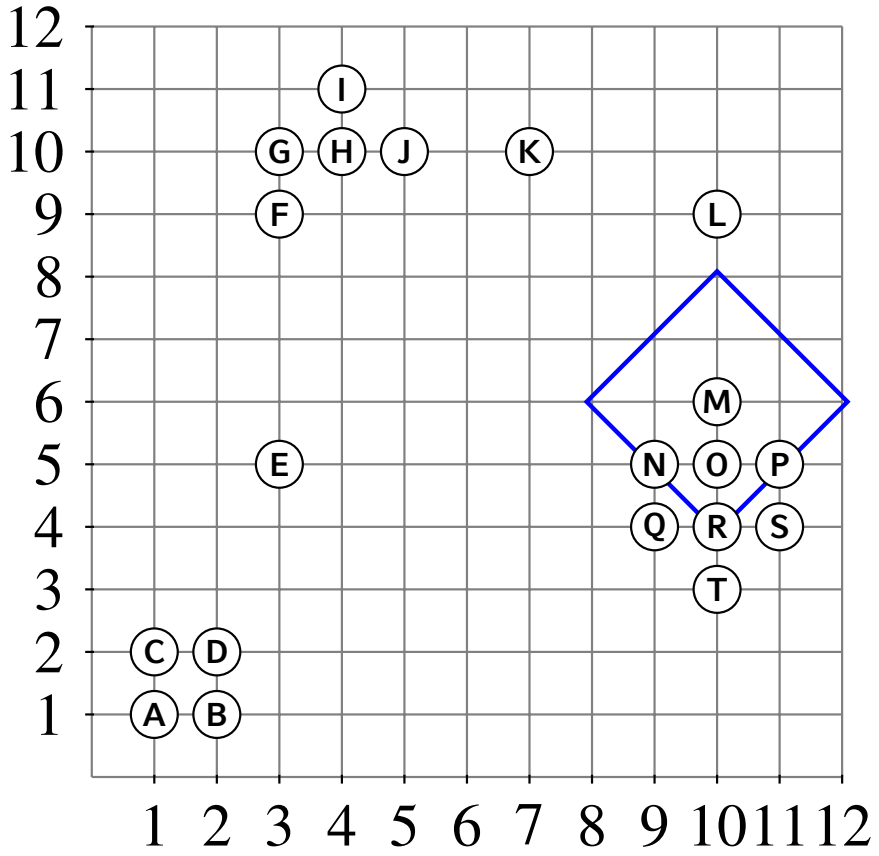
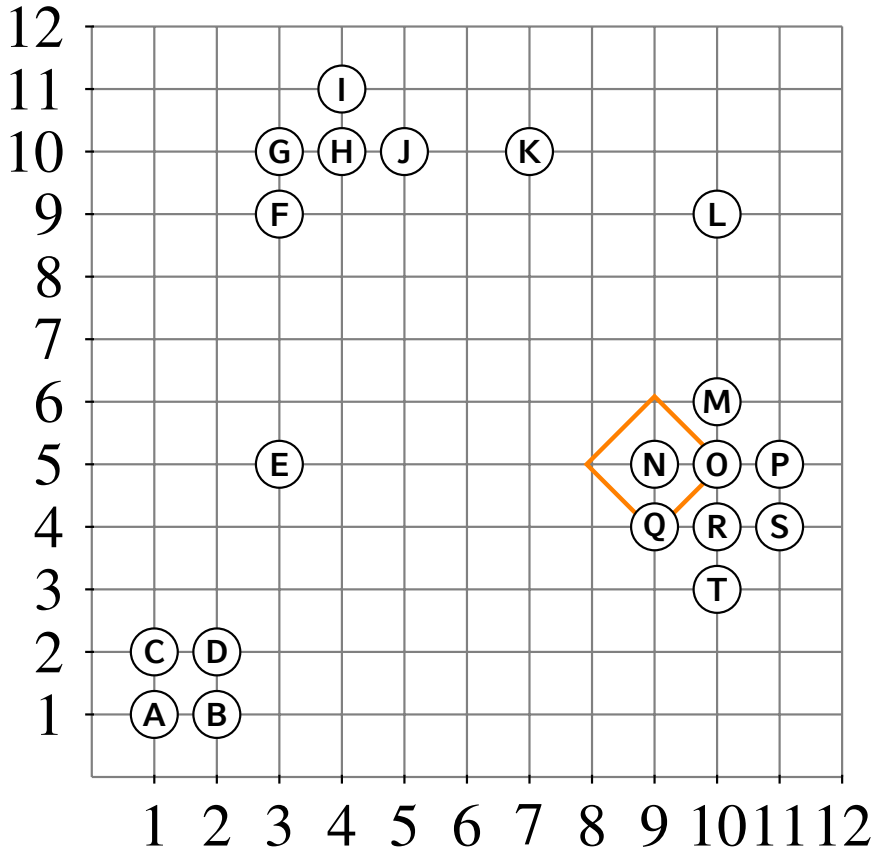
D	B C	1	A B C E	4
E	D F	4	B C D F G	5
F	G H	2	G H I J	3
G	F H	1	F H I J	2
H	G I J	1	F G I J	2
I	G H J	2	F G H J	3
J	G H I K	2	G H I K	2
K	H J	3	G H I J L	4
L	K M O	4	K M N O P R	5
M				
N				
O				
P				
Q				
R				
S				
T				

LOF and kNN



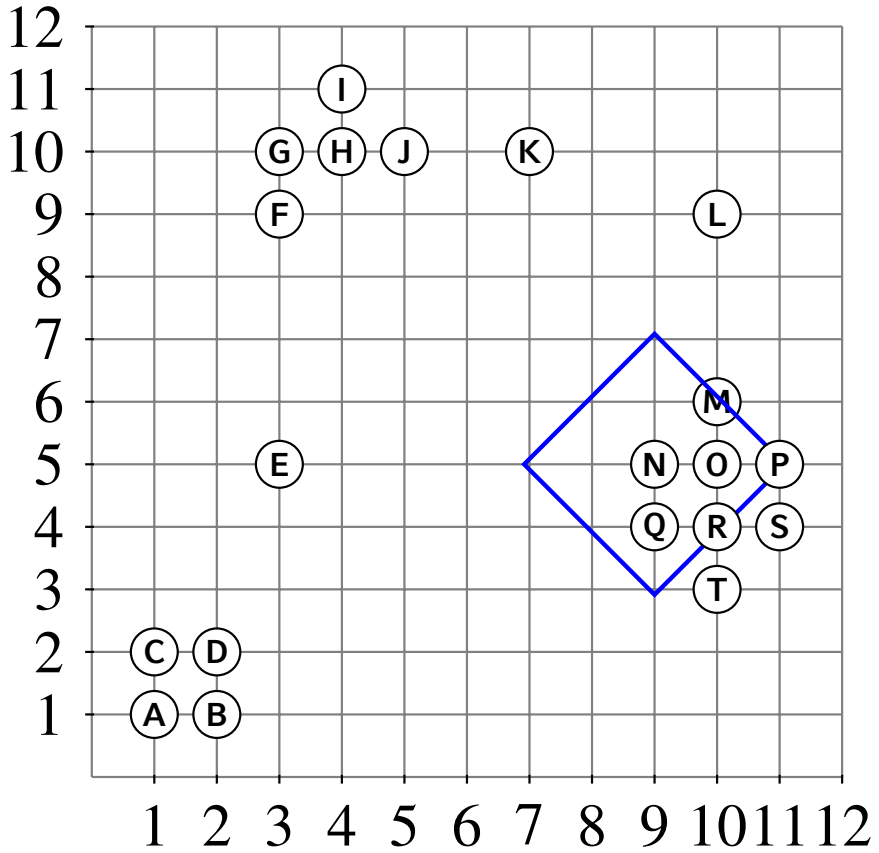
	2NN	2d.	4NN	4d.
A	B C	1	B C D E	6
B	A D	1	A C D E	5
C	A D	1	A B D E	5
D	B C	1	A B C E	4
E	D F	4	B C D F G	5
F	G H	2	G H I J	3
G	F H	1	F H I J	2
H	G I J	1	F G I J	2
I	G H J	2	F G H J	3
J	G H I K	2	G H I K	2
K	H J	3	G H I J L	4
L	K M O	4	K M N O P R	5
M	N O P R	2	N O P R	2
N				
O				
P				
Q				
R				
S				
T				

LOF and kNN



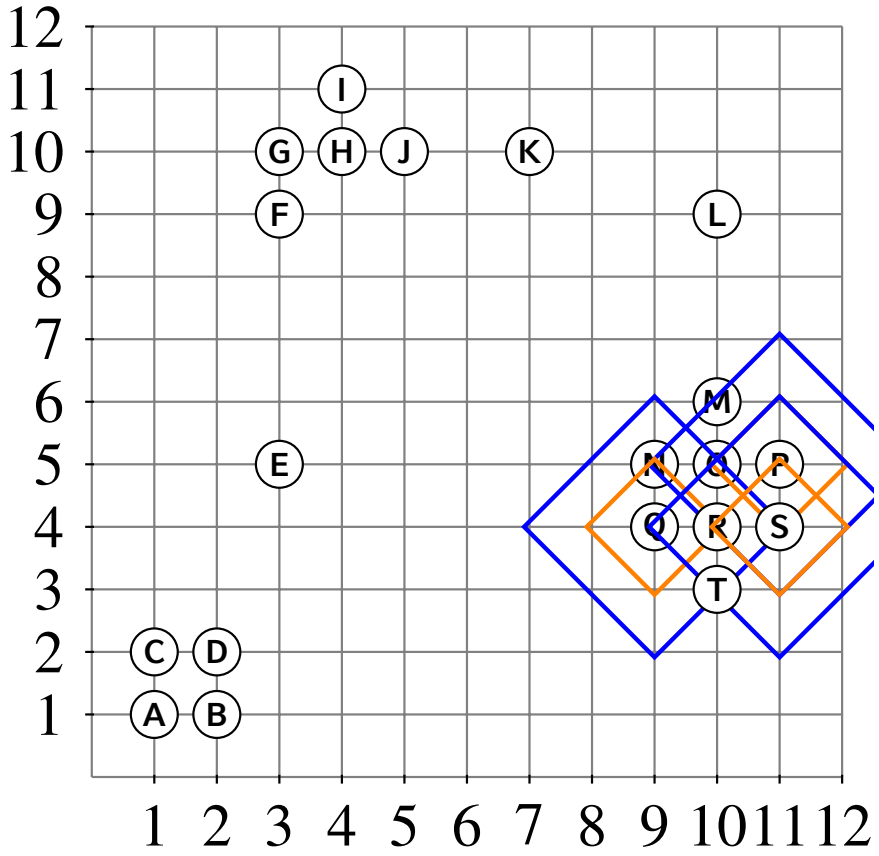
	2NN	2d.	4NN	4d.
A	B C	1	B C D E	6
B	A D	1	A C D E	5
C	A D	1	A B D E	5
D	B C	1	A B C E	4
E	D F	4	B C D F G	5
F	G H	2	G H I J	3
G	F H	1	F H I J	2
H	G I J	1	F G I J	2
I	G H J	2	F G H J	3
J	G H I K	2	G H I K	2
K	H J	3	G H I J L	4
L	K M O	4	K M N O P R	5
M	N O P R	2	N O P R	2
N	O Q	1		
O				
P				
Q				
R				
S				
T				

LOF and kNN



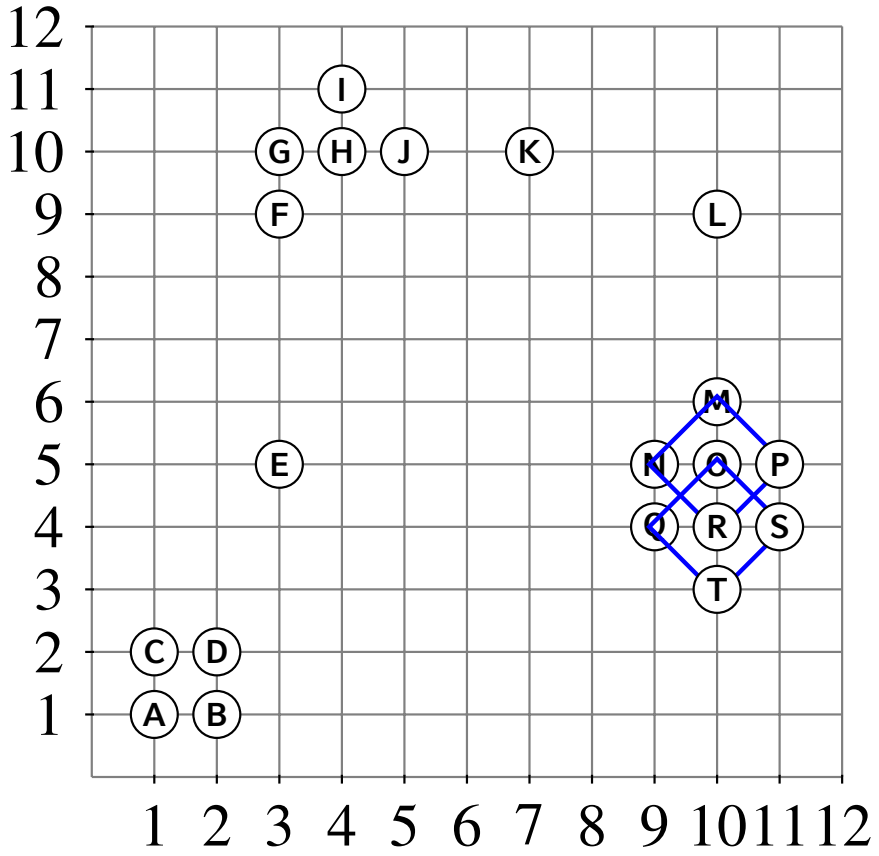
	2NN	2d.	4NN	4d.
A	B C	1	B C D E	6
B	A D	1	A C D E	5
C	A D	1	A B D E	5
D	B C	1	A B C E	4
E	D F	4	B C D F G	5
F	G H	2	G H I J	3
G	F H	1	F H I J	2
H	G I J	1	F G I J	2
I	G H J	2	F G H J	3
J	G H I K	2	G H I K	2
K	H J	3	G H I J L	4
L	K M O	4	K M N O P R	5
M	N O P R	2	N O P R	2
N	O Q	1	M O P Q R	2
O				
P				
Q				
R				
S				
T				

LOF and kNN



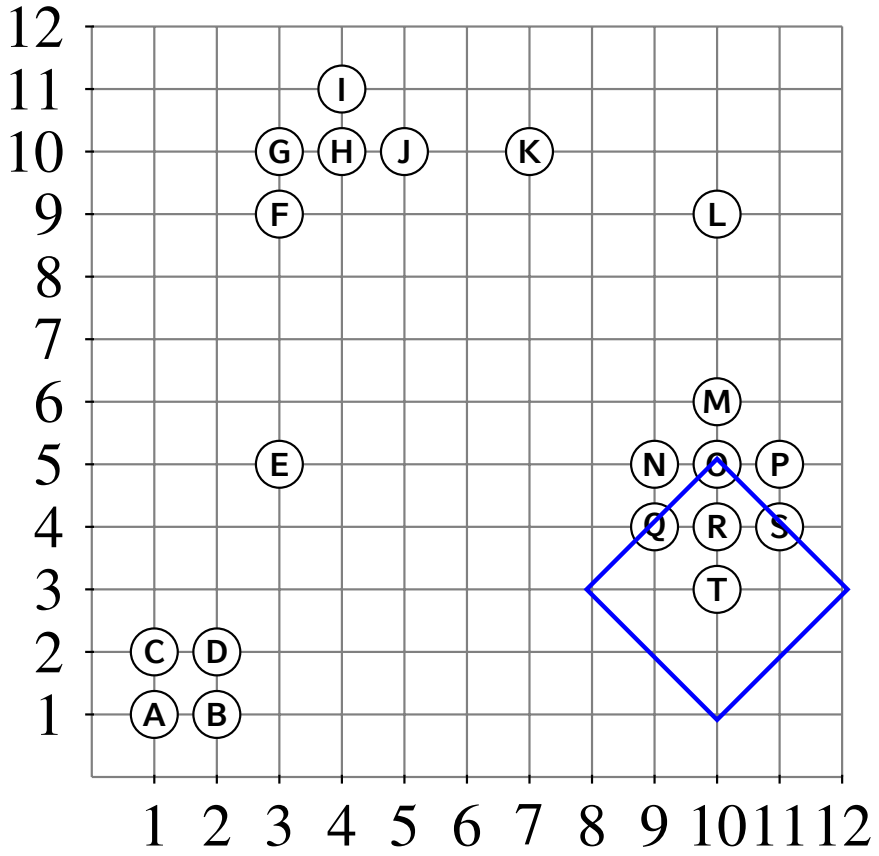
	2NN	2d.	4NN	4d.
A	B C	1	B C D E	6
B	A D	1	A C D E	5
C	A D	1	A B D E	5
D	B C	1	A B C E	4
E	D F	4	B C D F G	5
F	G H	2	G H I J	3
G	F H	1	F H I J	2
H	G I J	1	F G I J	2
I	G H J	2	F G H J	3
J	G H I K	2	G H I K	2
K	H J	3	G H I J L	4
L	K M O	4	K M N O P R	5
M	N O P R	2	N O P R	2
N	O Q	1	M O P Q R	2
O				
P	O S	1	M N O R S	2
Q	N R	1	N O R S T	2
R				
S	P R	1	O P Q R T	2
T				

LOF and kNN



	2NN	2d.	4NN	4d.
A	B C	1	B C D E	6
B	A D	1	A C D E	5
C	A D	1	A B D E	5
D	B C	1	A B C E	4
E	D F	4	B C D F G	5
F	G H	2	G H I J	3
G	F H	1	F H I J	2
H	G I J	1	F G I J	2
I	G H J	2	F G H J	3
J	G H I K	2	G H I K	2
K	H J	3	G H I J L	4
L	K M O	4	K M N O P R	5
M	N O P R	2	N O P R	2
N	O Q	1	M O P Q R	2
O	M N P R	1	M N P R	1
P	O S	1	M N O R S	2
Q	N R	1	N O R S T	2
R	O Q S T	1	O Q S T	1
S	P R	1	O P Q R T	2
T				

LOF and kNN



	2NN	2d.	4NN	4d.
A	B C	1	B C D E	6
B	A D	1	A C D E	5
C	A D	1	A B D E	5
D	B C	1	A B C E	4
E	D F	4	B C D F G	5
F	G H	2	G H I J	3
G	F H	1	F H I J	2
H	G I J	1	F G I J	2
I	G H J	2	F G H J	3
J	G H I K	2	G H I K	2
K	H J	3	G H I J L	4
L	K M O	4	K M N O P R	5
M	N O P R	2	N O P R	2
N	O Q	1	M O P Q R	2
O	M N P R	1	M N P R	1
P	O S	1	M N O R S	2
Q	N R	1	N O R S T	2
R	O Q S T	1	O Q S T	1
S	P R	1	O P Q R T	2
T	O Q R S	2	O Q R S	2

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	
C	A D	1	A B D E	5	-	
D	B C	1	A B C E	4		
E	D F	4	B C D F G	5		
F	G H	2	G H I J	3		
G	F H	1	F H I J	2	-	
H	G I J	1	F G I J	2		
I	G H J	2	F G H J	3	-	
J	G H I K	2	G H I K	2		
K	H J	3	G H I J L	4		
L	K M O	4	K M N O P R	5	-	
M	N O P R	2	N O P R	2		
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1		
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	
C	A D	1	A B D E	5	-	
D	B C	1	A B C E	4		
E	D F	4	B C D F G	5	$(4+4)/2$	
F	G H	2	G H I J	3		
G	F H	1	F H I J	2	-	
H	G I J	1	F G I J	2		
I	G H J	2	F G H J	3	-	
J	G H I K	2	G H I K	2		
K	H J	3	G H I J L	4		
L	K M O	4	K M N O P R	5	-	
M	N O P R	2	N O P R	2		
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1		
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	
C	A D	1	A B D E	5	-	
D	B C	1	A B C E	4		
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3		
G	F H	1	F H I J	2	-	
H	G I J	1	F G I J	2		
I	G H J	2	F G H J	3	-	
J	G H I K	2	G H I K	2		
K	H J	3	G H I J L	4		
L	K M O	4	K M N O P R	5	-	
M	N O P R	2	N O P R	2		
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1		
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	
C	A D	1	A B D E	5	-	
D	B C	1	A B C E	4	$(1+1)/2$	
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3		
G	F H	1	F H I J	2	-	
H	G I J	1	F G I J	2		
I	G H J	2	F G H J	3	-	
J	G H I K	2	G H I K	2		
K	H J	3	G H I J L	4		
L	K M O	4	K M N O P R	5	-	
M	N O P R	2	N O P R	2		
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1		
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	
C	A D	1	A B D E	5	-	
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3		
G	F H	1	F H I J	2	-	
H	G I J	1	F G I J	2		
I	G H J	2	F G H J	3	-	
J	G H I K	2	G H I K	2		
K	H J	3	G H I J L	4		
L	K M O	4	K M N O P R	5	-	
M	N O P R	2	N O P R	2		
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1		
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	
C	A D	1	A B D E	5	-	
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	
G	F H	1	F H I J	2	-	
H	G I J	1	F G I J	2		
I	G H J	2	F G H J	3	-	
J	G H I K	2	G H I K	2		
K	H J	3	G H I J L	4		
L	K M O	4	K M N O P R	5	-	
M	N O P R	2	N O P R	2		
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1		
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	
C	A D	1	A B D E	5	-	
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	
H	G I J	1	F G I J	2		
I	G H J	2	F G H J	3	-	
J	G H I K	2	G H I K	2		
K	H J	3	G H I J L	4		
L	K M O	4	K M N O P R	5	-	
M	N O P R	2	N O P R	2		
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1		
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	$(6+5+4+5)/4$
C	A D	1	A B D E	5	-	
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	
H	G I J	1	F G I J	2		
I	G H J	2	F G H J	3	-	
J	G H I K	2	G H I K	2		
K	H J	3	G H I J L	4		
L	K M O	4	K M N O P R	5	-	
M	N O P R	2	N O P R	2		
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1		
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	$(6+5+4+5)/4$
C	A D	1	A B D E	5	-	$(6+5+4+5)/4$
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	
H	G I J	1	F G I J	2		
I	G H J	2	F G H J	3	-	
J	G H I K	2	G H I K	2		
K	H J	3	G H I J L	4		
L	K M O	4	K M N O P R	5	-	
M	N O P R	2	N O P R	2		
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1		
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	$(6+5+4+5)/4$
C	A D	1	A B D E	5	-	$(6+5+4+5)/4$
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	$(3+2+3+2)/4$
H	G I J	1	F G I J	2		
I	G H J	2	F G H J	3	-	
J	G H I K	2	G H I K	2		
K	H J	3	G H I J L	4		
L	K M O	4	K M N O P R	5	-	
M	N O P R	2	N O P R	2		
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1		
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	$(6+5+4+5)/4$
C	A D	1	A B D E	5	-	$(6+5+4+5)/4$
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	$(3+2+3+2)/4$
H	G I J	1	F G I J	2		
I	G H J	2	F G H J	3	-	
J	G H I K	2	G H I K	2		
K	H J	3	G H I J L	4	$(2+3)/2$	
L	K M O	4	K M N O P R	5	-	
M	N O P R	2	N O P R	2		
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1		
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	$(6+5+4+5)/4$
C	A D	1	A B D E	5	-	$(6+5+4+5)/4$
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	$(3+2+3+2)/4$
H	G I J	1	F G I J	2		
I	G H J	2	F G H J	3	-	
J	G H I K	2	G H I K	2		
K	H J	3	G H I J L	4	$(2+3)/2$	$(4+3+4+2+5)/5$
L	K M O	4	K M N O P R	5	-	
M	N O P R	2	N O P R	2		
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1		
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	$(6+5+4+5)/4$
C	A D	1	A B D E	5	-	$(6+5+4+5)/4$
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	$(3+2+3+2)/4$
H	G I J	1	F G I J	2	$(1+2+2)/3$	
I	G H J	2	F G H J	3	-	
J	G H I K	2	G H I K	2		
K	H J	3	G H I J L	4	$(2+3)/2$	$(4+3+4+2+5)/5$
L	K M O	4	K M N O P R	5	-	
M	N O P R	2	N O P R	2		
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1		
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	$(6+5+4+5)/4$
C	A D	1	A B D E	5	-	$(6+5+4+5)/4$
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	$(3+2+3+2)/4$
H	G I J	1	F G I J	2	$(1+2+2)/3$	$(3+2+3+2)/4$
I	G H J	2	F G H J	3	-	
J	G H I K	2	G H I K	2		
K	H J	3	G H I J L	4	$(2+3)/2$	$(4+3+4+2+5)/5$
L	K M O	4	K M N O P R	5	-	
M	N O P R	2	N O P R	2		
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1		
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	$(6+5+4+5)/4$
C	A D	1	A B D E	5	-	$(6+5+4+5)/4$
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	$(3+2+3+2)/4$
H	G I J	1	F G I J	2	$(1+2+2)/3$	$(3+2+3+2)/4$
I	G H J	2	F G H J	3	-	
J	G H I K	2	G H I K	2	$(2+1+2+3)/4$	
K	H J	3	G H I J L	4	$(2+3)/2$	$(4+3+4+2+5)/5$
L	K M O	4	K M N O P R	5	-	
M	N O P R	2	N O P R	2		
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1		
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	$(6+5+4+5)/4$
C	A D	1	A B D E	5	-	$(6+5+4+5)/4$
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	$(3+2+3+2)/4$
H	G I J	1	F G I J	2	$(1+2+2)/3$	$(3+2+3+2)/4$
I	G H J	2	F G H J	3	-	
J	G H I K	2	G H I K	2	$(2+1+2+3)/4$	$(2+2+3+4)/4$
K	H J	3	G H I J L	4	$(2+3)/2$	$(4+3+4+2+5)/5$
L	K M O	4	K M N O P R	5	-	
M	N O P R	2	N O P R	2		
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1		
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	$(6+5+4+5)/4$
C	A D	1	A B D E	5	-	$(6+5+4+5)/4$
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	$(3+2+3+2)/4$
H	G I J	1	F G I J	2	$(1+2+2)/3$	$(3+2+3+2)/4$
I	G H J	2	F G H J	3	-	$(3+2+2+2)/4$
J	G H I K	2	G H I K	2	$(2+1+2+3)/4$	$(2+2+3+4)/4$
K	H J	3	G H I J L	4	$(2+3)/2$	$(4+3+4+2+5)/5$
L	K M O	4	K M N O P R	5	-	
M	N O P R	2	N O P R	2		
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1		
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	$(6+5+4+5)/4$
C	A D	1	A B D E	5	-	$(6+5+4+5)/4$
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	$(3+2+3+2)/4$
H	G I J	1	F G I J	2	$(1+2+2)/3$	$(3+2+3+2)/4$
I	G H J	2	F G H J	3	-	$(3+2+2+2)/4$
J	G H I K	2	G H I K	2	$(2+1+2+3)/4$	$(2+2+3+4)/4$
K	H J	3	G H I J L	4	$(2+3)/2$	$(4+3+4+2+5)/5$
L	K M O	4	K M N O P R	5	-	$(4+3+5+4+5+5)/6$
M	N O P R	2	N O P R	2		
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1		
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	$(6+5+4+5)/4$
C	A D	1	A B D E	5	-	$(6+5+4+5)/4$
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	$(3+2+3+2)/4$
H	G I J	1	F G I J	2	$(1+2+2)/3$	$(3+2+3+2)/4$
I	G H J	2	F G H J	3	-	$(3+2+2+2)/4$
J	G H I K	2	G H I K	2	$(2+1+2+3)/4$	$(2+2+3+4)/4$
K	H J	3	G H I J L	4	$(2+3)/2$	$(4+3+4+2+5)/5$
L	K M O	4	K M N O P R	5	-	$(4+3+5+4+5+5)/6$
M	N O P R	2	N O P R	2		
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1	$(2+1+1+1)/4$	
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	$(6+5+4+5)/4$
C	A D	1	A B D E	5	-	$(6+5+4+5)/4$
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	$(3+2+3+2)/4$
H	G I J	1	F G I J	2	$(1+2+2)/3$	$(3+2+3+2)/4$
I	G H J	2	F G H J	3	-	$(3+2+2+2)/4$
J	G H I K	2	G H I K	2	$(2+1+2+3)/4$	$(2+2+3+4)/4$
K	H J	3	G H I J L	4	$(2+3)/2$	$(4+3+4+2+5)/5$
L	K M O	4	K M N O P R	5	-	$(4+3+5+4+5+5)/6$
M	N O P R	2	N O P R	2		
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1	$(2+1+1+1)/4$	$(2+2+2+1)/4$
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	$(6+5+4+5)/4$
C	A D	1	A B D E	5	-	$(6+5+4+5)/4$
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	$(3+2+3+2)/4$
H	G I J	1	F G I J	2	$(1+2+2)/3$	$(3+2+3+2)/4$
I	G H J	2	F G H J	3	-	$(3+2+2+2)/4$
J	G H I K	2	G H I K	2	$(2+1+2+3)/4$	$(2+2+3+4)/4$
K	H J	3	G H I J L	4	$(2+3)/2$	$(4+3+4+2+5)/5$
L	K M O	4	K M N O P R	5	-	$(4+3+5+4+5+5)/6$
M	N O P R	2	N O P R	2	$(2+2+1+2)/4$	
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1	$(2+1+1+1)/4$	$(2+2+2+1)/4$
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	$(6+5+4+5)/4$
C	A D	1	A B D E	5	-	$(6+5+4+5)/4$
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	$(3+2+3+2)/4$
H	G I J	1	F G I J	2	$(1+2+2)/3$	$(3+2+3+2)/4$
I	G H J	2	F G H J	3	-	$(3+2+2+2)/4$
J	G H I K	2	G H I K	2	$(2+1+2+3)/4$	$(2+2+3+4)/4$
K	H J	3	G H I J L	4	$(2+3)/2$	$(4+3+4+2+5)/5$
L	K M O	4	K M N O P R	5	-	$(4+3+5+4+5+5)/6$
M	N O P R	2	N O P R	2	$(2+2+1+2)/4$	$(2+1+2+2)/4$
N	O Q	1	M O P Q R	2		
O	M N P R	1	M N P R	1	$(2+1+1+1)/4$	$(2+2+2+1)/4$
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	$(6+5+4+5)/4$
C	A D	1	A B D E	5	-	$(6+5+4+5)/4$
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	$(3+2+3+2)/4$
H	G I J	1	F G I J	2	$(1+2+2)/3$	$(3+2+3+2)/4$
I	G H J	2	F G H J	3	-	$(3+2+2+2)/4$
J	G H I K	2	G H I K	2	$(2+1+2+3)/4$	$(2+2+3+4)/4$
K	H J	3	G H I J L	4	$(2+3)/2$	$(4+3+4+2+5)/5$
L	K M O	4	K M N O P R	5	-	$(4+3+5+4+5+5)/6$
M	N O P R	2	N O P R	2	$(2+2+1+2)/4$	$(2+1+2+2)/4$
N	O Q	1	M O P Q R	2	$(1+1)/2$	
O	M N P R	1	M N P R	1	$(2+1+1+1)/4$	$(2+2+2+1)/4$
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	$(6+5+4+5)/4$
C	A D	1	A B D E	5	-	$(6+5+4+5)/4$
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	$(3+2+3+2)/4$
H	G I J	1	F G I J	2	$(1+2+2)/3$	$(3+2+3+2)/4$
I	G H J	2	F G H J	3	-	$(3+2+2+2)/4$
J	G H I K	2	G H I K	2	$(2+1+2+3)/4$	$(2+2+3+4)/4$
K	H J	3	G H I J L	4	$(2+3)/2$	$(4+3+4+2+5)/5$
L	K M O	4	K M N O P R	5	-	$(4+3+5+4+5+5)/6$
M	N O P R	2	N O P R	2	$(2+2+1+2)/4$	$(2+1+2+2)/4$
N	O Q	1	M O P Q R	2	$(1+1)/2$	$(2+1+2+2+2)/5$
O	M N P R	1	M N P R	1	$(2+1+1+1)/4$	$(2+2+2+1)/4$
P	O S	1	M N O R S	2		
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	$(6+5+4+5)/4$
C	A D	1	A B D E	5	-	$(6+5+4+5)/4$
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	$(3+2+3+2)/4$
H	G I J	1	F G I J	2	$(1+2+2)/3$	$(3+2+3+2)/4$
I	G H J	2	F G H J	3	-	$(3+2+2+2)/4$
J	G H I K	2	G H I K	2	$(2+1+2+3)/4$	$(2+2+3+4)/4$
K	H J	3	G H I J L	4	$(2+3)/2$	$(4+3+4+2+5)/5$
L	K M O	4	K M N O P R	5	-	$(4+3+5+4+5+5)/6$
M	N O P R	2	N O P R	2	$(2+2+1+2)/4$	$(2+1+2+2)/4$
N	O Q	1	M O P Q R	2	$(1+1)/2$	$(2+1+2+2+2)/5$
O	M N P R	1	M N P R	1	$(2+1+1+1)/4$	$(2+2+2+1)/4$
P	O S	1	M N O R S	2	$(1+1)/2$	
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	$(6+5+4+5)/4$
C	A D	1	A B D E	5	-	$(6+5+4+5)/4$
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	$(3+2+3+2)/4$
H	G I J	1	F G I J	2	$(1+2+2)/3$	$(3+2+3+2)/4$
I	G H J	2	F G H J	3	-	$(3+2+2+2)/4$
J	G H I K	2	G H I K	2	$(2+1+2+3)/4$	$(2+2+3+4)/4$
K	H J	3	G H I J L	4	$(2+3)/2$	$(4+3+4+2+5)/5$
L	K M O	4	K M N O P R	5	-	$(4+3+5+4+5+5)/6$
M	N O P R	2	N O P R	2	$(2+2+1+2)/4$	$(2+1+2+2)/4$
N	O Q	1	M O P Q R	2	$(1+1)/2$	$(2+1+2+2+2)/5$
O	M N P R	1	M N P R	1	$(2+1+1+1)/4$	$(2+2+2+1)/4$
P	O S	1	M N O R S	2	$(1+1)/2$	$(2+2+1+2+2)/5$
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1		
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	$(6+5+4+5)/4$
C	A D	1	A B D E	5	-	$(6+5+4+5)/4$
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	$(3+2+3+2)/4$
H	G I J	1	F G I J	2	$(1+2+2)/3$	$(3+2+3+2)/4$
I	G H J	2	F G H J	3	-	$(3+2+2+2)/4$
J	G H I K	2	G H I K	2	$(2+1+2+3)/4$	$(2+2+3+4)/4$
K	H J	3	G H I J L	4	$(2+3)/2$	$(4+3+4+2+5)/5$
L	K M O	4	K M N O P R	5	-	$(4+3+5+4+5+5)/6$
M	N O P R	2	N O P R	2	$(2+2+1+2)/4$	$(2+1+2+2)/4$
N	O Q	1	M O P Q R	2	$(1+1)/2$	$(2+1+2+2+2)/5$
O	M N P R	1	M N P R	1	$(2+1+1+1)/4$	$(2+2+2+1)/4$
P	O S	1	M N O R S	2	$(1+1)/2$	$(2+2+1+2+2)/5$
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1	$(1+1+1+2)/4$	
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

	2NN	2d.	4NN	4d.	$1/lrd_2$	$1/lrd_4$
A	B C	1	B C D E	6	-	-
B	A D	1	A C D E	5	-	$(6+5+4+5)/4$
C	A D	1	A B D E	5	-	$(6+5+4+5)/4$
D	B C	1	A B C E	4	$(1+1)/2$	$(6+5+5+5)/4$
E	D F	4	B C D F G	5	$(4+4)/2$	$(5+5+4+4+5)/5$
F	G H	2	G H I J	3	$(1+2)/2$	$(2+2+3+3)/4$
G	F H	1	F H I J	2	-	$(3+2+3+2)/4$
H	G I J	1	F G I J	2	$(1+2+2)/3$	$(3+2+3+2)/4$
I	G H J	2	F G H J	3	-	$(3+2+2+2)/4$
J	G H I K	2	G H I K	2	$(2+1+2+3)/4$	$(2+2+3+4)/4$
K	H J	3	G H I J L	4	$(2+3)/2$	$(4+3+4+2+5)/5$
L	K M O	4	K M N O P R	5	-	$(4+3+5+4+5+5)/6$
M	N O P R	2	N O P R	2	$(2+2+1+2)/4$	$(2+1+2+2)/4$
N	O Q	1	M O P Q R	2	$(1+1)/2$	$(2+1+2+2+2)/5$
O	M N P R	1	M N P R	1	$(2+1+1+1)/4$	$(2+2+2+1)/4$
P	O S	1	M N O R S	2	$(1+1)/2$	$(2+2+1+2+2)/5$
Q	N R	1	N O R S T	2	-	-
R	O Q S T	1	O Q S T	1	$(1+1+1+2)/4$	$(1+2+2+2)/4$
S	P R	1	O P Q R T	2	-	-
T	O Q R S	2	O Q R S	2	-	-

LOF and kNN

Compute the final LOF scores for $k = 2$:

$$LOF_2(E) := \frac{\frac{2}{2} + \frac{2}{3}}{2} / \frac{2}{8}$$
$$\approx 3.333$$
$$LOF_2(O) := \frac{\frac{4}{7} + \frac{2}{2} + \frac{2}{2} + \frac{4}{5}}{4} / \frac{4}{5}$$
$$\approx 1.054$$
$$LOF_2(K) := \frac{\frac{3}{5} + \frac{4}{9}}{2} / \frac{2}{5}$$
$$\approx 1.375$$

LOF and kNN

Compute the final LOF scores for $k = 4$:

$$LOF_4(E) := \frac{\frac{4}{20} + \frac{4}{20} + \frac{4}{21} + \frac{4}{10} + \frac{4}{10}}{5} / \frac{5}{23}$$
$$\approx 1.279$$

$$LOF_4(O) := \frac{\frac{4}{7} + \frac{5}{9} + \frac{5}{9} + \frac{4}{7}}{4} / \frac{4}{7}$$
$$\approx 0.986$$

$$LOF_4(K) := \frac{\frac{4}{10} + \frac{4}{10} + \frac{4}{9} + \frac{4}{11} + \frac{6}{26}}{5} / \frac{5}{18}$$
$$\approx 1.324$$

LOF and kNN

Compute the final LOF scores for $k = 4$:

$$LOF_4(E) := \frac{\frac{4}{20} + \frac{4}{20} + \frac{4}{21} + \frac{4}{10} + \frac{4}{10}}{5} \Big/ \frac{5}{23}$$
$$\approx 1.279$$

$$LOF_4(O) := \frac{\frac{4}{7} + \frac{5}{9} + \frac{5}{9} + \frac{4}{7}}{4} \Big/ \frac{4}{7}$$
$$\approx 0.986$$

$$LOF_4(K) := \frac{\frac{4}{10} + \frac{4}{10} + \frac{4}{9} + \frac{4}{11} + \frac{6}{26}}{5} \Big/ \frac{5}{18}$$
$$\approx 1.324$$

$|\mathcal{N}|$ of candidate – $|\mathcal{N}|$ of neighbors – reachdist sums

LOF and kNN

Both LOF and kNN suffer from $k = 4$ being the size of the small cluster $\{A, B, C, D\}$.

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But it remains one of the highest scoring outliers, and A, B, C, D remain inliers!

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Both LOF and kNN suffer from $k = 4$ being the size of the small cluster $\{A, B, C, D\}$.

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But it remains one of the highest scoring outliers, and A, B, C, D remain inliers!

$\Rightarrow$ performance of LOF degrades less with a bad k.