3 Baumstrukturen zur Sekundärschlüsselsuche

Ziel: Unterstützung von Anfragen über mehrere Attribute
(Sekundärschlüsselsuche = Multiattributssuche).

3.1 Invertierte Listen

In fast allen kommerziell vertriebenen Datenbanksystemen:

Multiattributssuche mit Hilfe ***invertierter Listen***.

- ***Primärindex***
  Index über den Primärschlüssel.
- ***Sekundärindex***
  Index über ein Attribut, das kein Primärschlüssel ist.

  Im Gegensatz zu einem Primärindex beeinflußt der Sekundärindex den Ort der Speicherung eines Datensatzes nicht. Es werden nur Verweise gespeichert.

Konzept der invertierten Listen:

- Für anfragerelevante Attribute werden Sekundärindizes ***(invertierte Listen)*** angelegt.

$\Rightarrow$ Damit steht für jedes relevante Attribut eine eindimensionale Indexstruktur zur Verfügung.

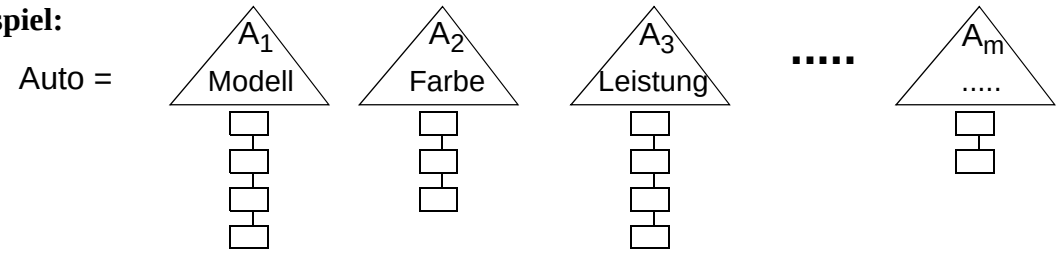
Multiattributssuche für invertierte Listen:

Eine Anfrage spezifiziere die Attribute A_1, ... ,A_m:

- *m Anfragen über m Indexstrukturen*

  Ergebnis:
  m Listen mit Verweisen auf die entsprechenden Antwortkandidaten in der Datei.
- *Mengentheoretische Verknüpfung* (z.B. Durchschnitt) der m Listen gemäß der Anfrage.

Beispiel:



Anfrage: Gesucht sind alle Autos mit Modell = GLD, Farbe = rot und Leistung = 75 PS

Eigenschaften:

- Die Antwortzeit ist nicht proportional zur Anzahl der Antworten.
- Die Suche dauert umso länger, je mehr Attribute spezifiziert sind.

  Ursache für beide Beobachtungen:
  Die Attributswerte eines Datensatzes sind nicht in einer Struktur miteinander verbunden.
- Invertierte Listen sind einigermaßen effizient, wenn die Antwortlisten sehr klein sind.
- Invertierte Listen haben hohe Kosten für Update-Operationen.
- Sekundärindizes beeinflußen die physische Speicherung der Datensätze nicht.

  $\Rightarrow$ Ordnungserhaltung über den Sekundärschlüssel nicht möglich.

  $\Rightarrow$ schlechtes Leistungsverhalten von invertierten Listen.

3.2 Hierarchie von B-Bäumen: MDB-Bäume

Ziel: Speicherung multidimensionaler Schlüssel (a_k, ... , a_1) in einer Indexstruktur.

Idee: Hierarchie von Bäumen**,** wobei jede Hierarchiestufe jeweils einem Attribut entspricht.

Notation:

Im folgenden werden für eine einfachere Notation die Attributswerte multidimensionaler Schlüssel absteigend numeriert (a_k, ... ,a_1).

Außerdem ändert sich die Numerierung der Höhen: Blattknoten eines Baumes haben die Höhe 1 und die Höhe der Wurzel entspricht der Höhe des Baumes.

Multidimensionale B-Bäume (MDB-Bäume) [SO 82]

- k-stufige Hierarchie von B-Bäumen.
- Jede Hierarchiestufe ***(Level)*** entspricht einem Attribut.
  Werte des Attributs a_i werden in Level i ($1 \le i \le k$) gespeichert.
- Die B-Bäume des Levels i haben die *Ordnung* m_i;
  m_i hängt von der Länge der Werte des i-ten Attributs ab.

Verzeigerung in einem B-Baum:

- linker Teilbaum bzgl. a_i: ***LOSON(a_i)***
- rechter Teilbaum bzgl. a_i: ***HISON(a_i)***

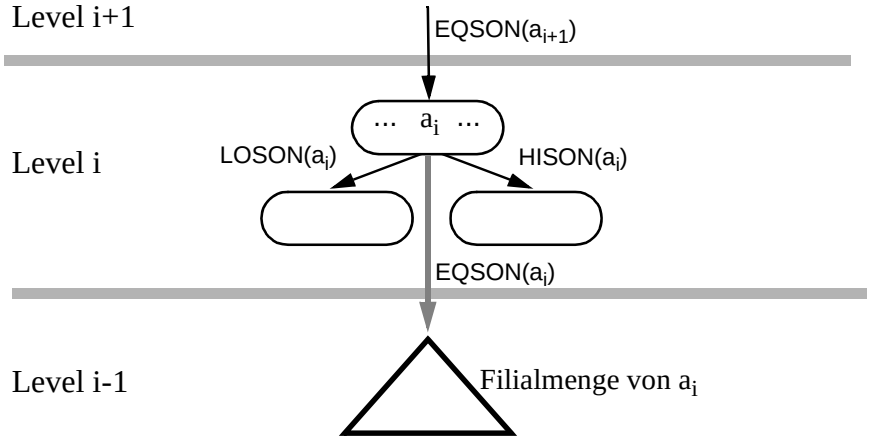
Verknüpfung der Level:

- Jeder Attributswert a_i hat zusätzlich einen *EQSON-Zeiger*:
  EQSON(a_i) zeigt auf den B-Baum des Levels i-1, der die verschiedenen Werte abspeichert, die zu Schlüsseln mit Attributswert a_i gehören.

  Filialmenge

  Für einen *Präfix* (a_k, ... , a_i) eines Schlüssels (a_k, ... , a_1), i < k, betrachte man die Menge M aller Schlüssel in der Datei mit diesem Präfix. Die Menge der (i-1)-dimensionalen Schlüssel, die man aus M durch Weglassen des gemeinsamen Präfixes erhält, heißt *Filialmenge* von (a_k, ... , a_i) (oder kurz: Filialmenge von a_i).

$\Rightarrow$ EQSON(a_i) zeigt auf einen B-Baum, der die Filialmenge von a_i abspeichert.



Exact Match Queries:
können effizient beantwortet werden.

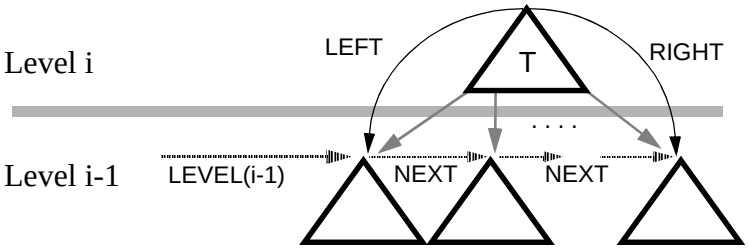
Range, Partial Match und Partial Range Queries:
erfordern zusätzlich sequentielle Verarbeitung auf jedem Level.

Unterstützung von Range Queries:

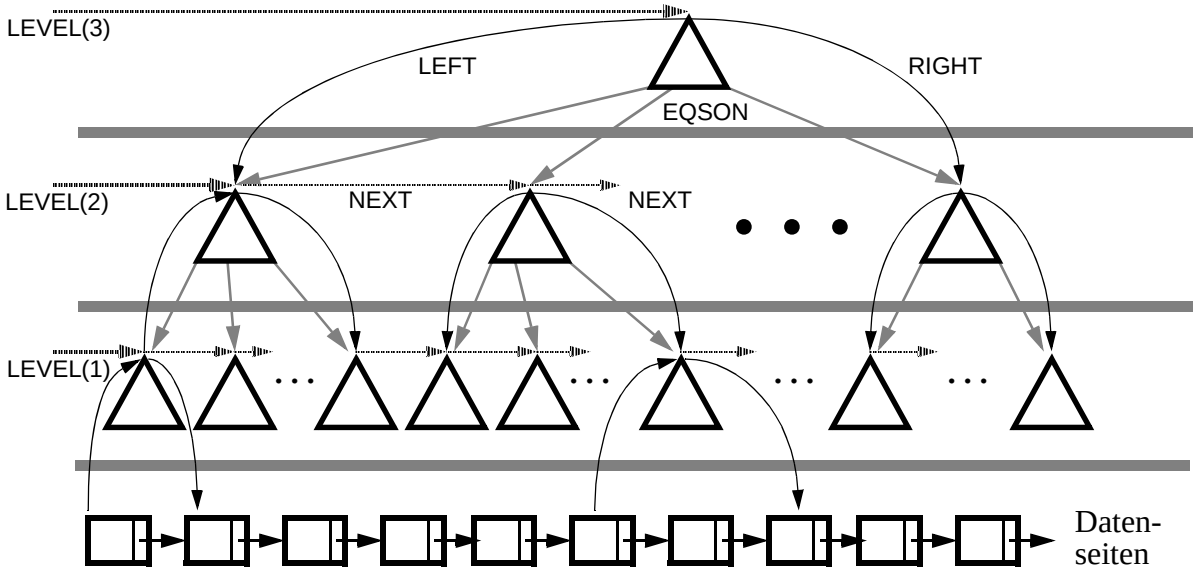
- *Verkettung der Wurzeln* der B-Bäume eines Levels: ***NEXT-Zeiger***.

Unterstützung von Partial Match Queries und Partial Range Queries:

- *Einstiegszeiger für jedes Level:*
  - ***LEVEL(i)*** zeigt für Level i auf den Beginn der verketteten Liste aus NEXT-Zeigern
- *Überspringzeiger* von jeder Level-Wurzel eines Level-Baumes T zum folgenden Level:
  - ***LEFT(T)*** zeigt zur Filialmenge des kleinsten Schlüssels von T
  - ***RIGHT(T)*** zeigt zur Filialmenge des größten Schlüssels von T.



Beispiel: MDB-Baum



Höhenabschätzung

Annahme: Für jedes Attribut gilt:

- Die Werte sind gleichverteilt im zugehörigen Wertebereich.
- Die Werte eines Attributs sind unabhängig von den Werten anderer Attribute *(stochastische Unabhängigkeit der Attribute)*

⇒ Alle Bäume auf demselben Level haben dieselbe Höhe.

⇒ Die maximale Höhe ist O (log N + k).

Einwand: Die Annahmen sind in realen Dateien äußerst selten erfüllt.

Die maximale Höhe eines MDB-Baumes, der die obige Annahme nicht erfüllt, beträgt:

O (k*log N)

3.3 Balancierung über die Attribute hinweg: kB-Bäume

Ziel:

MDB-Baum, der eine maximale Höhe von log N + k unabhängig von der Verteilung der Daten garantiert

→ ***kB-Bäume*** ([GK 80], [Kri 82]).

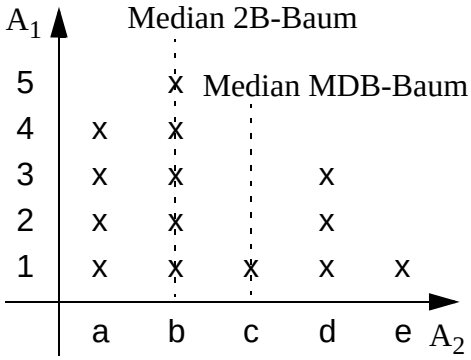
Konzept:

Balancierung über alle Attribute hinweg

⇒ auf höheren Leveln: *verzerrte B-Bäume*

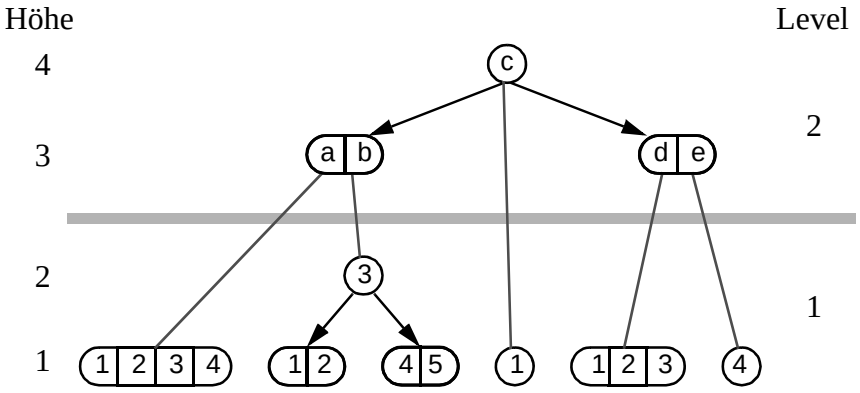
Beispiel:

Gegeben seien folgende zweidimensionalen Datensätze (a_2,a_1)
(als Punkte in der Ebene im folgenden Diagramm):

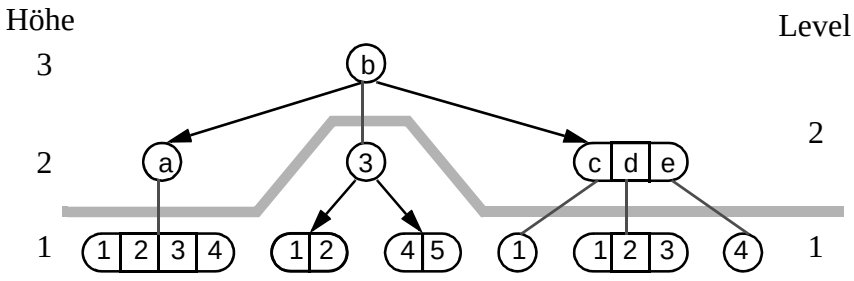


Speicherung dieser Daten:

- 2-Level MDB-Baum der Ordnung 2:



- kB-Baum der Ordnung 2 (k=2):

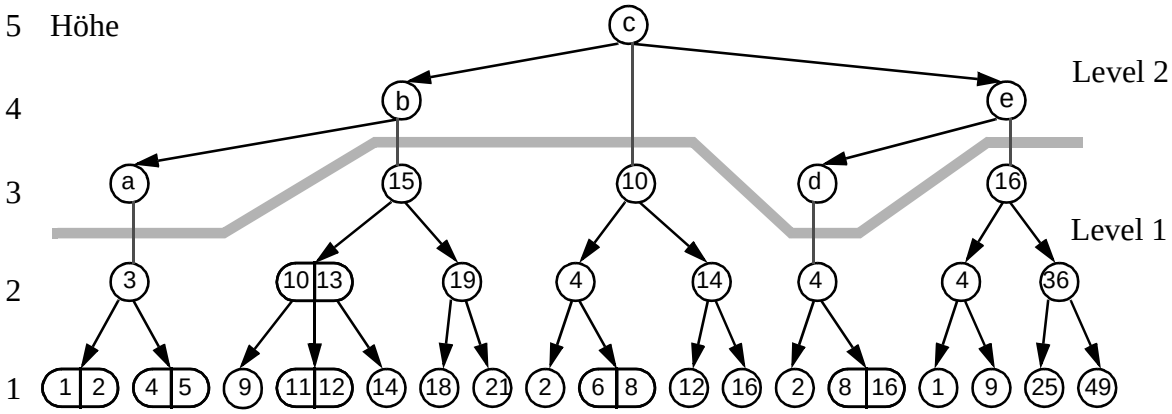


Realisierung des Balancierens

- über Eigenschaften der Attributswerte (im folgenden auch als Schlüssel bezeichnet):
  - Auf dem Level des 1. Attributs haben wir normale B-Bäume.
  - Jeder Attributswert (Schlüssel) eines anderen Levels wird aufgrund der Höhe seines EQSON-Teilbaumes positioniert.
  - Jeder Schlüssel ist entweder *kB-Repräsentant* oder *kB-Separator* (auch kurz *Repräsentant* oder *Separator* genannt):
    - ***kB-Repräsentant***
      - Die Höhe ist gleich der Höhe des EQSON-Teilbaumes plus 1.
      - Ein kB-Repräsentant ist *nicht absenkbar*.
      - Repräsentanten haben die niedrigst mögliche Höhe.
    - ***kB-Separator***
      - Die Höhe ist größer als die Höhe des EQSON-Teilbaumes plus 1.
      - Ein kB-Separator ist *absenkbar*.
      - Alle Söhne des kB-Separators sind auf allen Höhen, auf die er absenkbar ist, mindestens minimal gefüllt, d.h. zu mindestens 50 %.
- Durch die Eigenschaft, daß jeder Schlüssel entweder Separator oder Repräsentant ist, rechtfertigen die Schlüssel ihre Höhe.

Beispiel: 2B-Baum der Ordnung 1 für die Datensätze (a_2,a_1):

(a,1), (a,2), (a,3), (a,4), (a,5),
(b,9), (b,10), (b,11), (b,12), (b,13), (b,14), (b,15), (b,18), (b,19), (b,21),
(c,2), (c,4), (c,6), (c,8), (c,10), (c,12), (c,14), (c,16),
(d,2), (d,4), (d,8), (d,16),
(e,1), (e,4), (e,9), (e,16), (e,25), (e,36), (e,49)



- Auf Level 1: *normale B-Bäume*.
- Auf Level 2: *verzerrter B-Baum*.
- Schlüssel a, b, d und e: *kB-Repräsentanten*.
- Schlüssel c: *kB-Separator*
  c hat Höhe 5, ist aber auf Höhe 4 absenkbar. Da seine Söhne auf Höhe 4 (LOSON und HISON) minimal gefüllt sind (mit jeweils 1 Schlüssel), kann c seine Höhe rechtfertigen, obwohl der Schlüssel nicht seine niedrigst mögliche Höhe hat.

Struktureigenschaften eines kB-Baumes der Ordnung m:

(i) Jeder Knoten enthält mindestens 1 Schlüssel und höchstens 2m Schlüssel desselben Attributs.

(ii) Jeder Schlüssel ist entweder kB-Separator oder kB-Repräsentant.

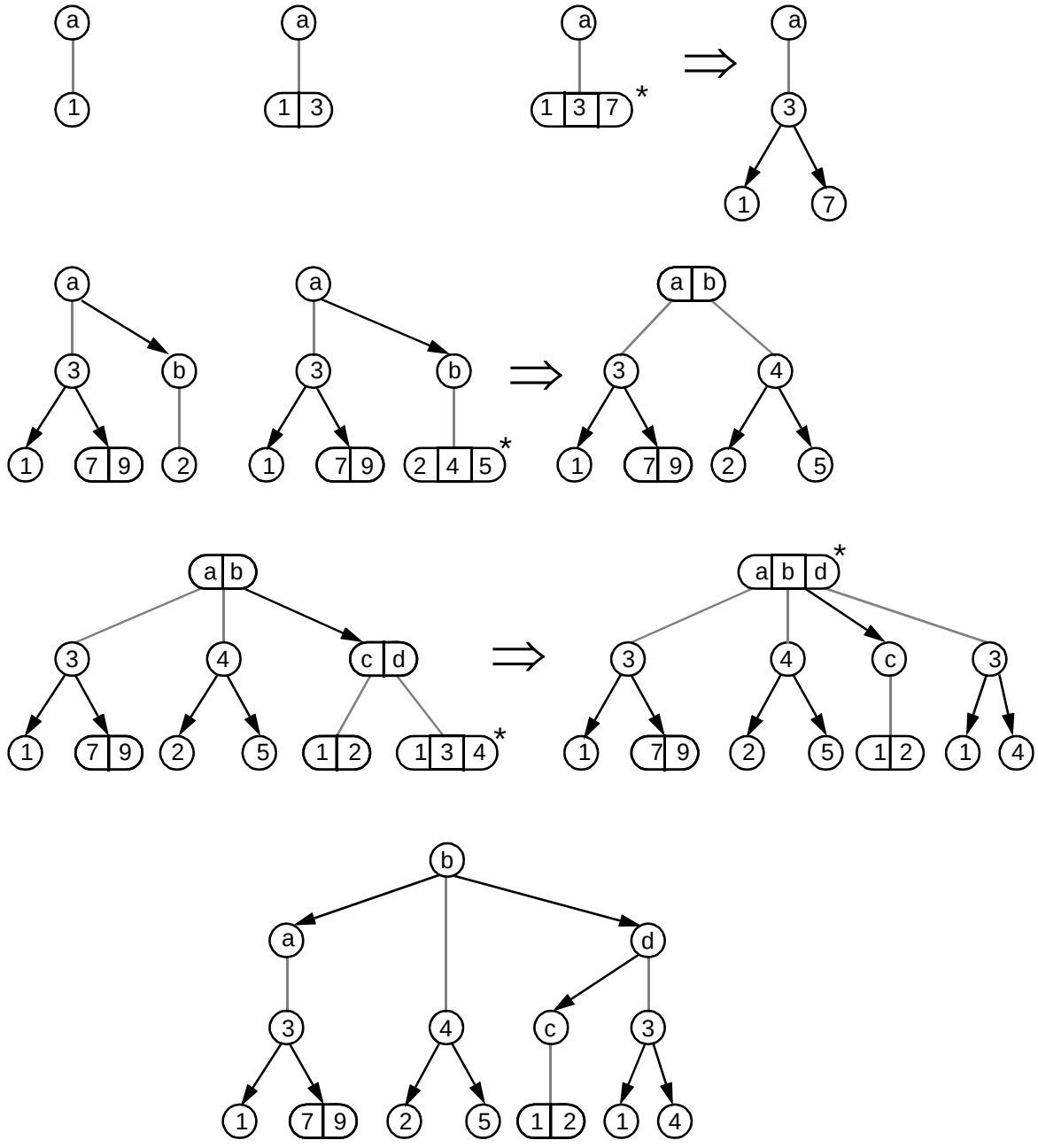
(ii) $\Rightarrow$ jeder Schlüssel ist so niedrig wie möglich.

Beispiel für einen 2B-Baum der Ordnung 1

Datensätze (in dieser Reihenfolge eingefügt):

(a,1), (a,3), (a,7), (a,9), (b,2), (b,4), (b,5), (c,1), (c,2), (d,1), (d,3), (d,4).

Einfügeprozeß:



Eigenschaften

- *kB-Bäume verallgemeinern B-Bäume auf den multidimensionalen Fall*

  Für jeden kB-Separator ist die Höhe seines LOSON-Teilbaumes gleich der Höhe seines HISON-Teilbaumes. Diese Eigenschaft des kB-Baumes ist eine Verallgemeinerung des Balancierens in B-Bäumen auf den multidimensionalen Fall, so daß alle Blätter den gleichen Abstand von der Wurzel haben.

- *kB-Bäume mit k = 1 sind normale B-Bäume.*

  Die Struktureigenschaften (i) und (ii) werden bei k=1 zu den üblichen B-Baum-Bedingungen:

  - Alle Schlüssel in den Blättern sind nicht absenkbar und daher kB-Repräsentanten.
  - Nach (ii) sind alle anderen Schlüssel kB-Separatoren. Da jeder Schlüssel auf Höhe 1 absenkbar ist, enthält jeder Knoten mit Ausnahme der Wurzel mindestens m Schlüssel. Die Wurzel enthält mindestens einen Schlüssel.
  - Nach (i) enthält jeder Knoten höchstens 2m Schlüssel.
  - Für jeden kB-Separator ist die Höhe seines LOSON-Teilbaumes gleich der Höhe seines HISON-Teilbaumes.

  Mit diesen Eigenschaften ist der 1B-Baum ein B-Baum.

- *Höhe von kB-Bäumen*

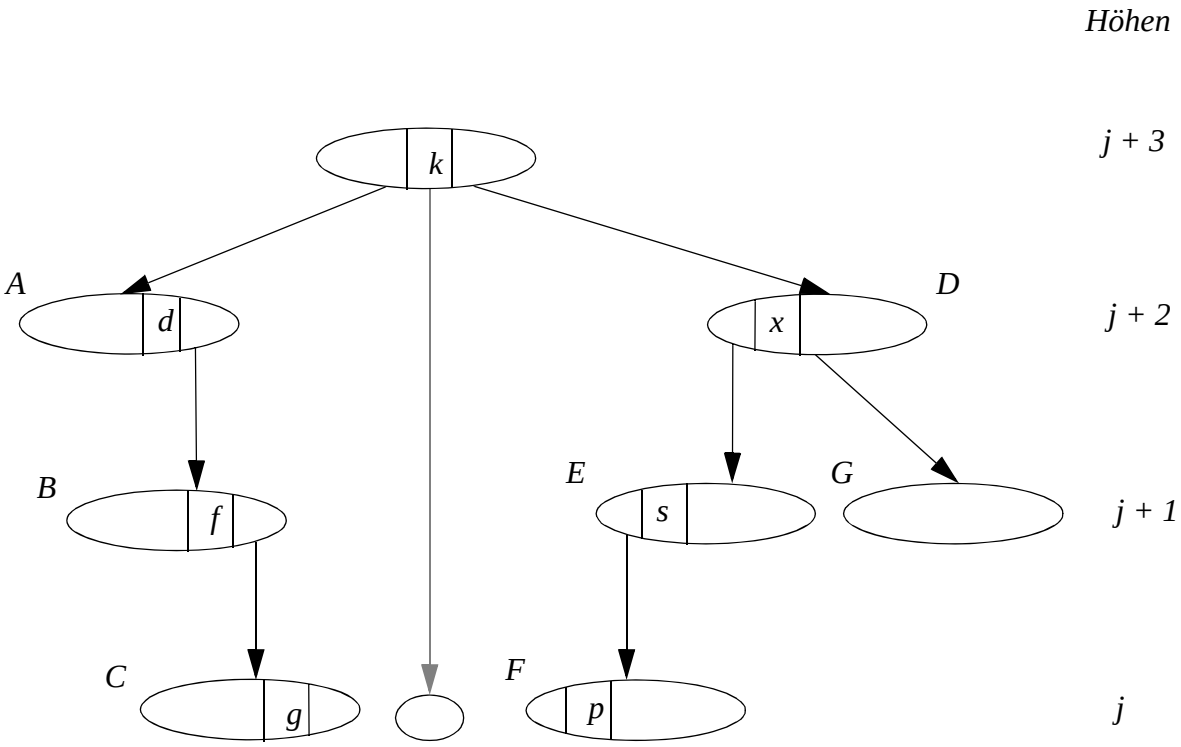
  Die Höhe eines kB-Baumes der Ordnung m mit N Datensätzen ist begrenzt durch:

  $$h \leq \log_{m+1}(N) + k.$$

Weitere Bezeichnungen

- Ein Knoten der Höhe h und des Levels i heißt *(h,i)-Knoten.*
- Ein Knoten heißt *Wurzel eines Levels,* wenn er keinen Vaterknoten auf demselben Level hat.
- Ein (h,i)-Knoten P heißt *Blatt des Levels i,* wenn h = i gilt.
  Diese Eigenschaft impliziert, daß P keine Söhne auf Level i hat.

Vaterschlüssel, Söhne und Brüder



- k ist *direkter rechter Vaterschlüssel* von Knoten A und *direkter linker Vaterschlüssel* von Knoten D.
- k ist *indirekter Vaterschlüssel* der Knoten B, C, E und F.
- Der Knoten B hat den *direkten linken Vaterschlüssel* d und den *indirekten rechten Vaterschlüssel* k.
- Die Knoten A, B, C, D, E und F sind *Söhne* von Schlüssel k;
  A und D sind *direkte Söhne* von k.
- A und D sind *direkte Brüder,*
  B und E sowie C und F sind *indirekte Brüder*.

3.4 Einfügealgorithmus des kB-Baumes

Remark:

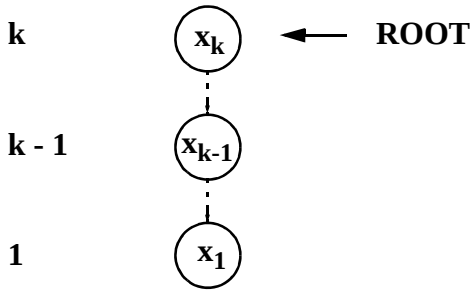
The following insertion algorithm assumes a kB-tree of order d.

Let $x = (x_k, x_{k-1}, \ldots, x_1)$ be the record which should be inserted. We assume that x is not stored in the tree.

Case 1: The tree is empty.

Then we create a node of height h = k containing key x_k, pointing with its EQSON pointer to a node of height k - 1 containing key x_{k-1}, etc. The leaf contains key x_1.
As a result we have the following chain of nodes.

height:



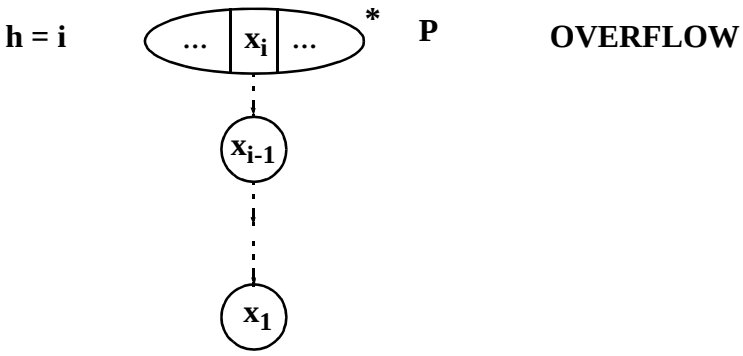
This structure is a correct kB-tree since all keys are representatives.

Case 2: The tree is not empty.

Searching for the keys x_k, x_{k-1}, … we traverse the levels k, k - 1, … . Since record x is not stored in the tree, we will reach some level i which does not contain key x_i. Thus, the search ends in an (h, i)-node P in which x_i falls between two keys.

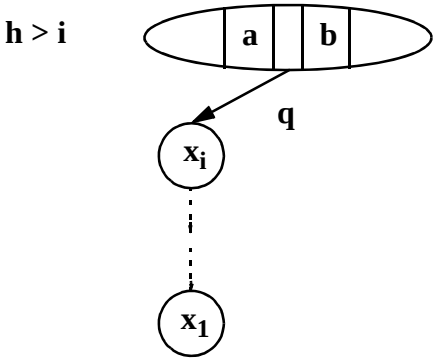
2.1.: P is a leaf of level i, i.e. h = i.

We insert key x_i in node P and create a chain of EQSON-nodes below key x_i storing the keys $x_{i-1}, \ldots, x_1$. As a result we have



Key x_i is a representative. The insertion of key x_i may have created an OVERFLOW in node P, denoted by the star. The restructuring will be described in the next section.

2.2.: P is not a leaf of level i, i.e. h > i, and the value of the pointer q which we follow from node P, is **nil**.
We create a chain of nodes storing x_i, …, x_1 and let q point to the root of this chain.



Since pointer q pointed to an empty and thus underfilled node, keys a and b are representatives. Thus the modified structure is correct. Since no OVERFLOW can occur, the insertion is finished.

Restructuring Operations

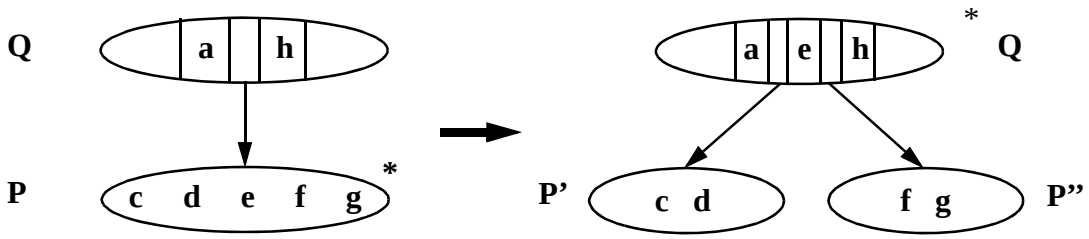
We will generalize the three restructuring operations which we know from B-trees: splitting a node, balancing of two brother nodes and collapsing two brother nodes. Additionally, we need two new operations: lift a key in a certain node up to its father and eliminate the root of a level.

1 SPLIT (P)

This operation is performed if an OVERFLOW occurs in node P. The middle key of the 2d + 1 keys in node P is pushed up into the father node and splits the overfilled nodes into two fragments of each d keys. Let P be an (h, i)-node.

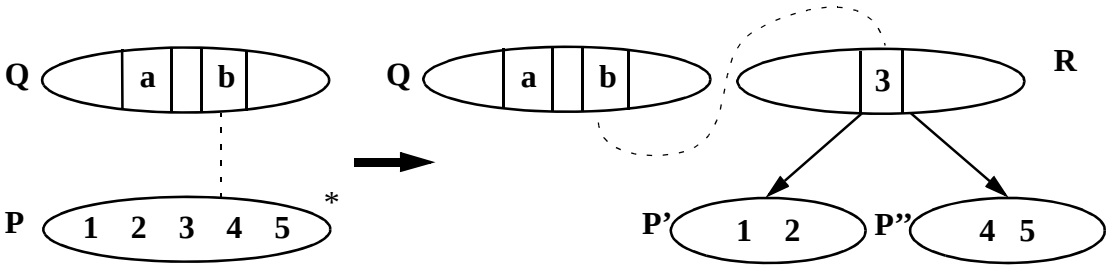
Case 1: The father of P has height h + 1.

1.1.: The father Q of P belongs to level i.



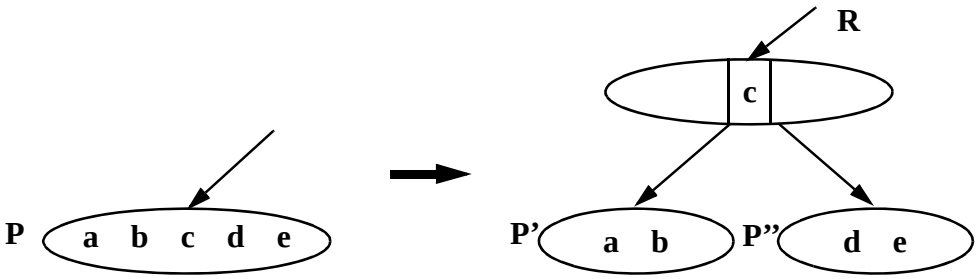
If Q now contains 2d + 1 keys, then SPLIT (Q).

1.2.: The father Q of P belongs to level i + 1.



and lift key b in Q: LIFTKEY (b, Q)

Case 2: The father of P has height > h + 1 or does not exist.



An operation is called correct if the result of the operation violates the structure properties (i) - (ii) of a kB-tree only at positions where further operations will be performed. For proving correctness of an operation, the following facts are helpful.

Fact 1: Condition (ii) can only be violated if
a) an underfilled node is created (separator) or
b) the root of a level is eliminated (separator or representative) or
c) the height of the EQSON-subtree increases (representative)

Fact 2: A node may be underfilled without violating the kB-tree structure if its left and right father key both either do not exist or are not sinkable to its height.

We will now verify that SPLIT is correct.

Correctness:

Case 1.1.: Since P is split in the middle, no underfilled node is created. Thus key e is separator on height h. All other keys keep their former separator or representative function.

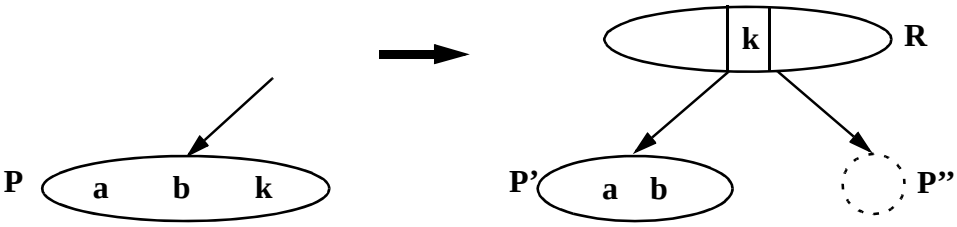
Case 1.2.: Since node R has no father key on the same level, it may be underfilled. Key 3 is separator. All other keys remain in their former separator or representative function. The violation of the representative property of b will be treated by LIFTKEY.

Case 2.: An underfilled node R containing key c is created. Since we had a correct situation, both father keys of R are representatives. Therefore, R may be underfilled. Key c is separator.

2 Liftkey (k, P)

This operation is initiated by case 1.2. in SPLIT. In node P some key k has to be lifted to the father of P. Thus node P is split into two parts of arbitrary size. Let P be an (h, i)-node.

Case 1.: The father of P has height > h + 1 or does not exist.

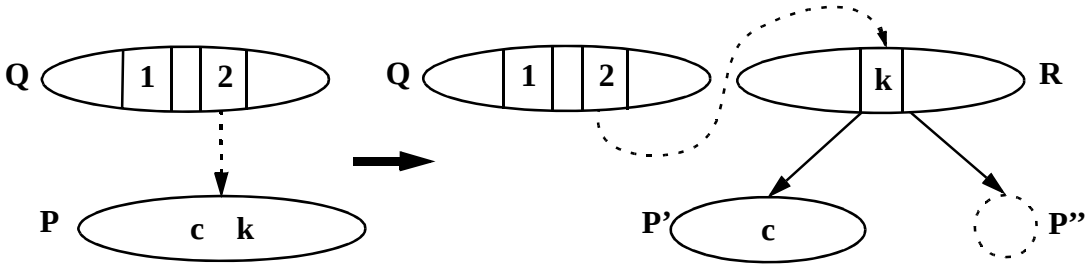


Correctness:

The father key of node R does not exist or is not sinkable. Key k is not sinkable. Thus nodes P', P'' and R may be underfilled.

Case 2.: The father of P has height h + 1.

2.1. The father Q of P belongs to level i + 1.

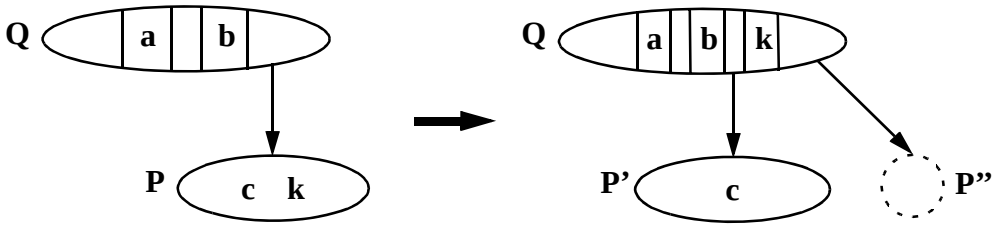


and LIFTKEY (2, Q)

Correctness:

Since for node R there is no sinkable father key of the same level and key k is not sinkable, nodes P', P'' and R may be underfilled.

2.2.: The father Q of P belongs to level i.



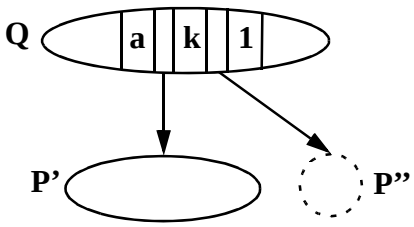
The following violations of the kB-tree structure may have been created by these transformations:

1. Q has too many keys, OVERFLOW.
2. P' is underfilled, the left father key of P' may have lost its separator property.
3. P'' is underfilled, the right father key of P'' may have lost its separator property.

In each case the situation can be corrected such that only one **OVERFLOW** or **UNDERFLOW** remains.

Case a: All father keys of P' and P'' are in Q.

h + 1



a.1 Both father keys a and 1 are representatives. UNDERFLOW in P’ and P’’ does not violate the kB-tree structure. Possible OVERFLOW in Q has to be treated.

a.2 Exactly one of P’ and P’’ is underfilled and the corresponding father key is sinkable.

a.2.1. Balancing with the direct (left in case of P’, right in case of P’’) brother is possible. As a result of balancing, the kB-tree conditions are satisfied below height h + 1. Possible OVERFLOW in Q has to be treated.

a.2.2. Collapsing with the direct brother is possible.
As a result of collapsing the kB-tree conditions are satisfied below height h + 1. Since the number of keys in Q is the same as before performing LIFTKEY (k, P), the restructuring is finished.

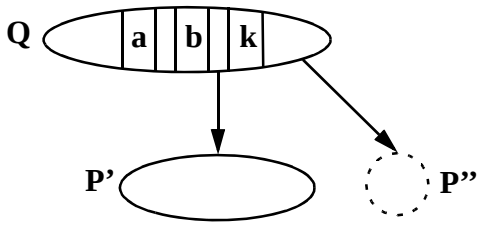
a.3 Both P’ and P’’ are underfilled and the left father key of P’ as well as the right father key of P’’ are sinkable.

a.3.1. For both P’ and P’’ balancing is possible.
Possible OVERFLOW in Q has to be treated.

a.3.2. For one of P’ and P’’ balancing is possible, for the other one collapsing is possible.
Since the number of keys in Q is the same as before performing LIFTKEY (k, P), the restructuring is finished.

a.3.3. For both P’ and P’’ collapsing is possible.
Since the number of keys in Q is one less than before performing LIFTKEY (k, P), possible UNDERFLOW in Q has to be treated.

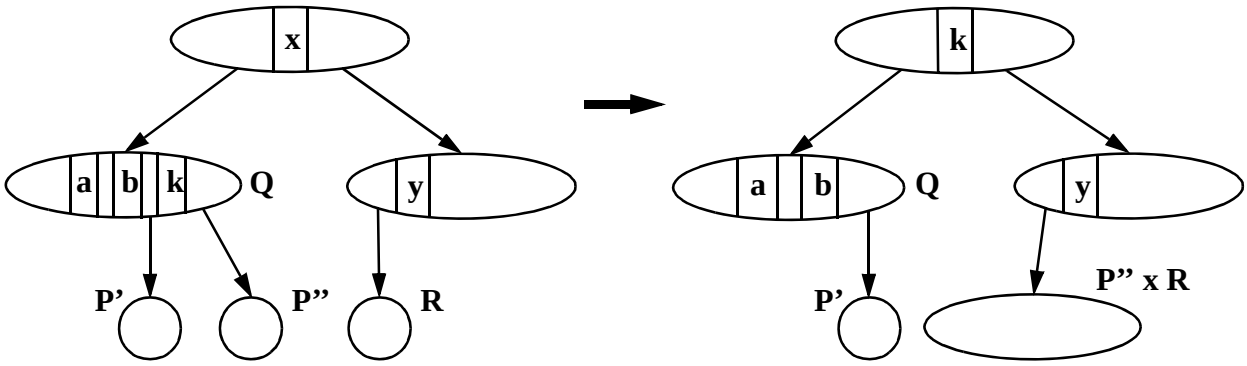
Case b: Exactly one father key except for key k is in Q. Let this be key b.



b.1 b and the indirect father key of P’’ are representatives. UNDERFLOW in P’ and P’’ does not violate the kB-tree structure. Possible OVERFLOW in Q has to be treated.

b.2 Collapsing of P’ with the direct left brother is possible.
As a result of collapsing, the number of keys in Q is the same as before performing LIFTKEY (k, P). If the right father key of P’’ lost its separator property, balancing or collapsing is performed. In case of collapsing an UNDERFLOW may occur.

b.3 b is a representative or P’ is balanced with the direct brother.
In both cases the number of keys in Q is one more than before performing LIFTKEY (k, P). If balancing with the indirect brother is possible for P’’, an OVERFLOW in Q remains. If balancing is not possible, P’’ is collapsed with its indirect brother in the following way:



Since this transformation leaves a correct kB-tree, restructuring is finished.

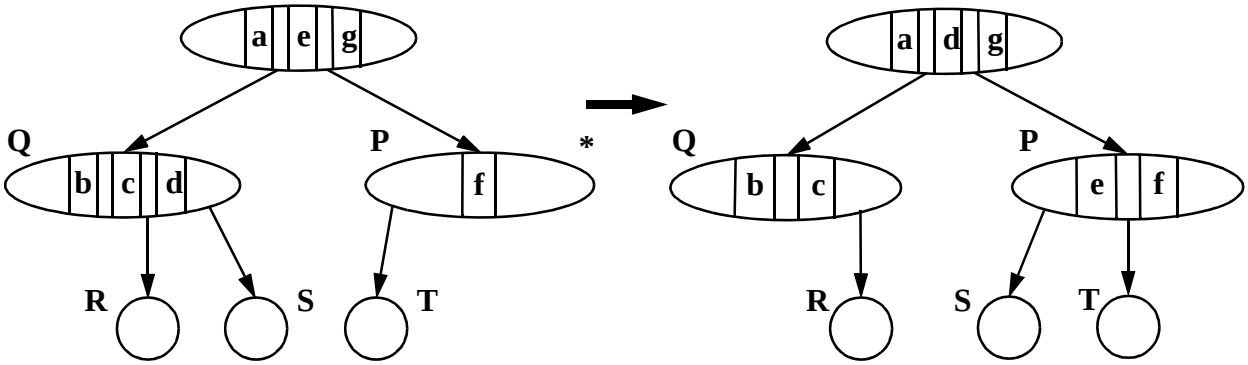
UNDERFLOW treatment

Balancing with a brother

Node P is balanced with its brother Q if P and Q together contain at least 2d keys. Otherwise P and Q are collapsed.

a. Balancing with a direct brother

Let P be the underfilled node which causes balancing.

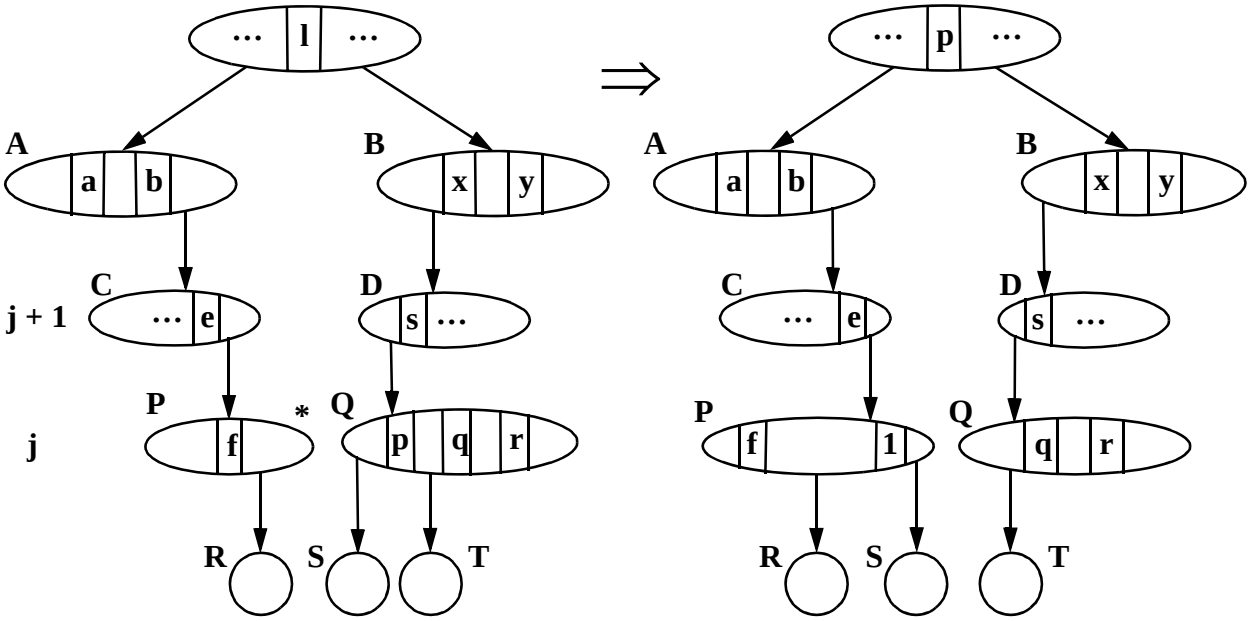


Balancing with a direct brother can be performed if the common father key of the two brothers, in our case key e, is sinkable. If none of the two father keys of P is sinkable, the situation is already correct.

Correctness:

The separator property of key e was violated only on height j. Thus nodes S and T and further indirect sons of key e have at least d keys or e is not sinkable to their height. Therefore, e fullfills condition (ii) in its new position. If the separator property of key g was violated, it is now repaired by filling up node P. In case key d was separator in its old position, d is sinkable to height j - 1. In either case, key d is separator on height j, since Q and P are not underfilled. Thus d is separator.

b. Balancing with an indirect brother:



Balancing with an indirect brother is applicable only if the indirect father key of node P, in our case key l, is sinkable to height j. If the direct father key of P, in our case key e, would be sinkable to height j, it would be better to balance with the direct left brother.

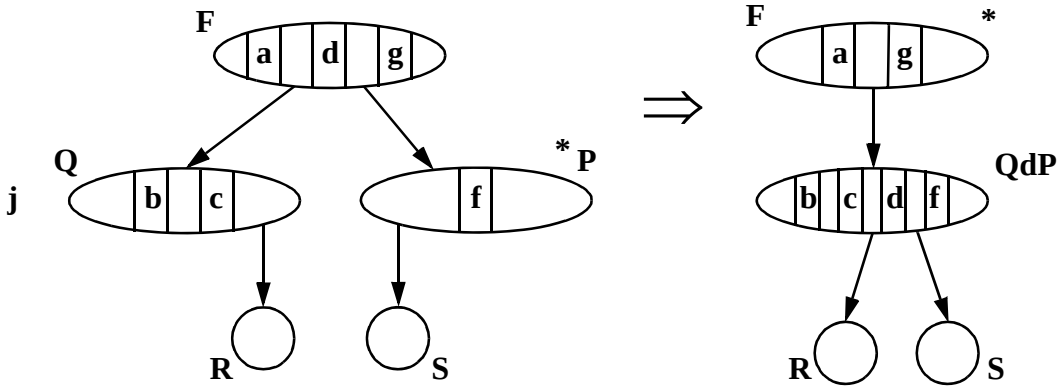
Correctness:

Node A - D are not underfilled, since key l was sinkable to their height. Thus key p is separator on heights > j. Nodes P and Q on height j are not underfilled any more and below height j, key p has the same sons as before. Therefore key p is separator. Key l fulfills condition (ii) since the tree fulfilled the kB-tree properties below height j.

Collapsing with a brother

The underfilled node P is collapsed with its brother Q if P and Q together contain less than 2d keys.

a. Collapsing with a direct brother

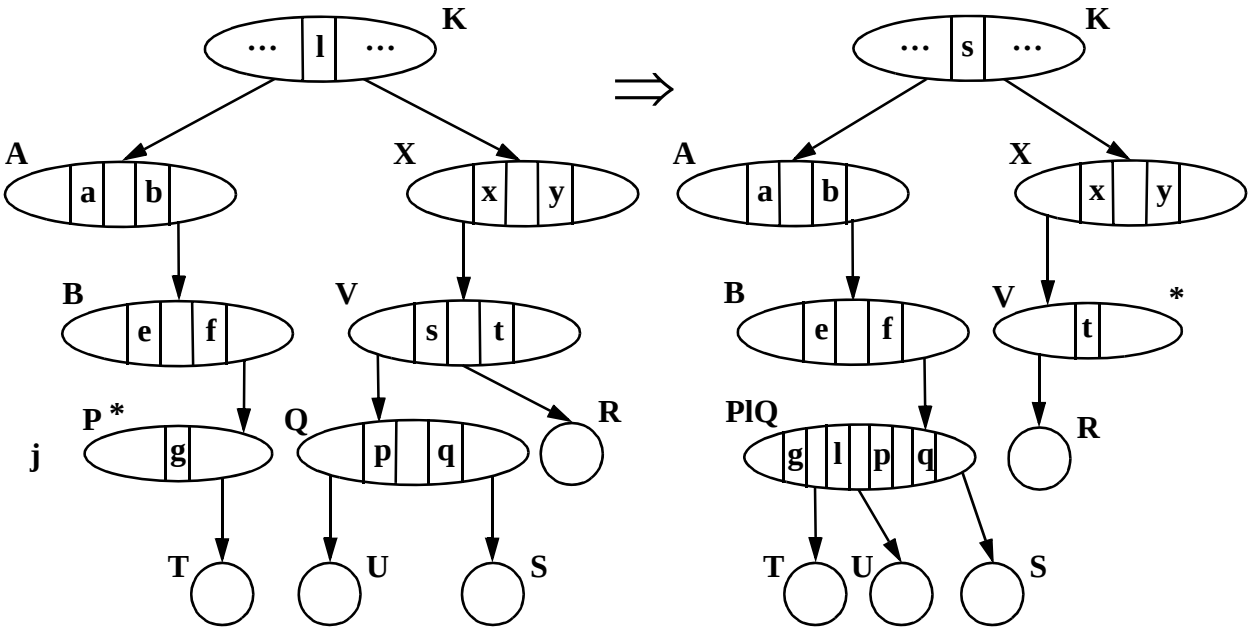


Collapsing with a direct brother is applicable if the common father key d of the nodes Q and P is sinkable to height j. If none of the two father keys of P is sinkable, the situation is already correct. An UNDERFLOW may occur in node F which will be treated.

Correctness:

Key d is separator below height j if it is sinkable, representative otherwise.

b. Collapsing with an indirect brother



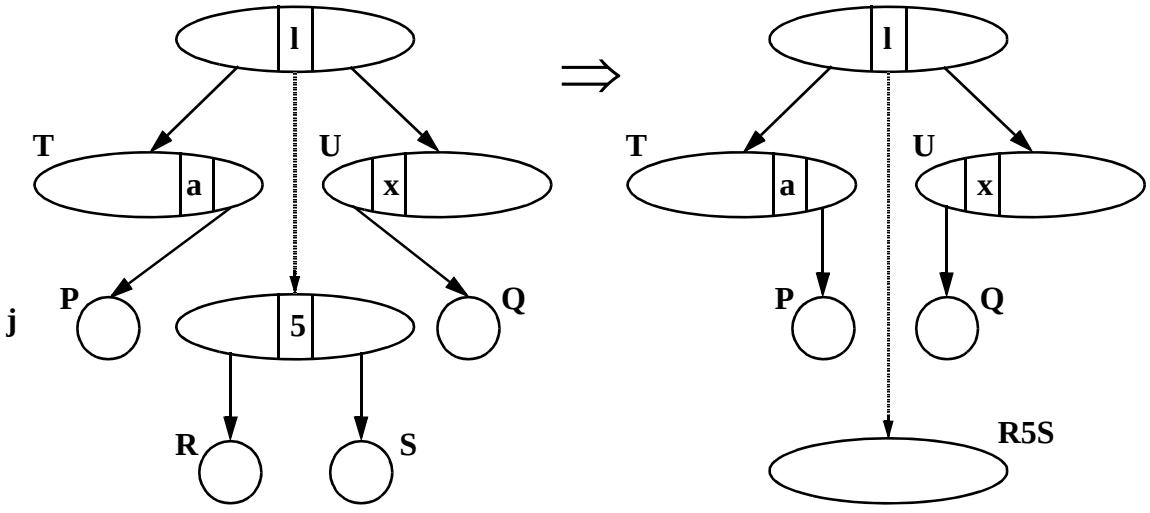
Collapsing with an indirect brother is possible if the indirect father key of node P, in our case key l, is sinkable to height j. An UNDERFLOW may occur in node V which will be treated.

Correctness:

In case key l is sinkable below height j, l is a separator, otherwise a representative. For key s the situation remains unchanged on height j and below. The sons of key s on heights j + 1 and j + 2 are not underfilled with the possible exception of node V which will be treated.

Eliminating the root of a level

The root of a level can be underfilled. If by collapsing its two sons, the last key is deleted from the root, the root is eliminated.



Thus, the father key l of this level is now sinkable to height j. Therefore, we have to check whether its two sons on height j, the nodes P and Q are underfilled. If this is the case, nodes P and Q are balanced or collapsed to a new node PlQ.

Correctness:

In case of collapsing P and Q, the collapsed node PlQ has key a as its left father key and key x as its indirect right father key. Thus a possible UNDERFLOW in U has to be treated. There is one crucial case. If both P and Q are underfilled, the collapsed node PlQ may still be underfilled. Then keys a and x, the two father keys of PlQ, are not sinkable to height j. Thus the underfilling of node PlQ does not violate the kB-tree structure. If exactly one of P and Q is underfilled, PlQ is not underfilled.

Now let us consider once more the insertion algorithm together with the restructuring operations. The restructuring operations are initiated only in case 2.1 of the insertion algorithm where a key is inserted in a leaf P of a level which may create an OVERFLOW in P. This is treated by calling SPLIT (P) which may create again an OVERFLOW (case 1.1 of SPLIT (P)). Thus SPLIT (P) may be called for the root of a level. If the result violates condition (ii), LIFTKEY has to be performed in the next level. As a result, OVERFLOW, UNDERFLOW or FINISH can occur. OVERFLOW is treated as already mentioned. In case of UNDERFLOW, we try to balance or collapse with direct or indirect brothers. If balancing is possible, the restructuring is finished. Collapsing may result in a further UNDERFLOW. If collapsing eliminates the root of a level, UNDERFLOW may occur in the next higher level. OVERFLOW and UNDERFLOW can walk up a path in the tree until finally reaching the root of the tree which may be split or eliminated.

What concerns the insertion time, the only problematic restructuring operation is collapsing with an indirect brother. Collapsing nodes P and Q (see diagram) may result in an UNDERFLOW of V and later A. In order to guarantee that not for each collapsing operation the indirect father key l and the indirect brother is determined again, walking up the search path, we bring the nodes P, B and A on a stack. Walking down from the indirect father key l, we stack the nodes pairwise, i.e. AX, BV and PQ. Thus the nodes of the search path and their brothers are visited at most once which is necessary for an external storage implementation. With this implementation of collapsing, insertion time is $O(\log_{(d+1)} N + k)$ in the worst case.