Lecture Notes for

Deep Learning and Artificial Intelligence

Winter Term 2018/2019

Value Function Approximation

DBS

Short Comings of the methods so far

So far: All methods work on a discrete state space S.

$\Rightarrow$ A policy π is a table of the form $\{(s_1, a_1), .., (s_{|S|}, a_{|S|})\}$

$\Rightarrow$ If we encounter a new state, we do not know what to do.

$\Rightarrow$ No matter how similar two states are, we learn Q(s,a) independently.

$\Rightarrow$ If |S| is very large:

- We need a lot of memory to store the policy.
- We need enormous amounts of samples to estimate Q(s,a) for all state-action pairs.

$\Rightarrow$ Previous models for MDPs and Reinforcement learning become infeasible

Some examples

- Number of states for some problems
  - Backgammon: 10^{20}
  - Computer Go: 10^{170}
  - Flying an RC Helicopter: continuous state space

Working with continuous State Spaces

Idea: What if we do not distinguish states but state descriptions, e.g., feature vectors?

- Depending on the feature space we can describe an infinite set of states. But some states might have the same description.

  => c.f. we often work on observations not states anyway

- A policy can be described as a function *f* of the state space
  - $\Rightarrow$ f(x,a) = Q(s,a) or f(x) = a
  - $\Rightarrow$ Mathematical functions are much more space efficient than tables
- State descriptions can be related to each other => if we do not have encountered a particular state description so far, we can derive a proper action from similar situations. (generalization)

Generally: Working on state descriptions allows for flexible agents being able to cope with unknown situations.

Overview on continuous State Spaces

- Value function approximation (this lecture)
  - Learn a function f to predict $U(x_s)$ or $Q(x_s,a)$
    (generally f is a regression function of some kind)

- Policy gradient methods: (next lecture)
  - Directly learn a function $f(x_s)$ predicting the best action a for x_s

- Actor Critic methods: (next lecture)
  - combine policy functions and value function approximation

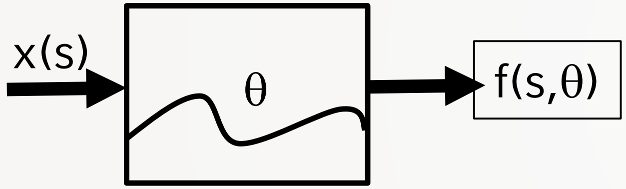
Value function approximation

Given: A mapping *x(s)* describing *s* in *IR*^{d}.

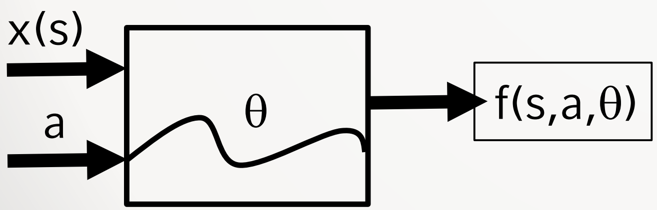
Idea: Learn a function that either describes the utility U(S) or the state-value function Q(S,A).

Options to learn the $f(s,\theta) \approx U(S)$ or $f(s,a,\theta) \approx Q(S,A)$:

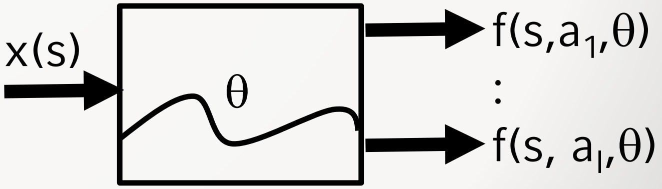
- Approximate U(*S)*



- *Approximate Q(S,A)*



or



Value Function Approximation and Partial Observability

A side-effect of using value function approximation is that we can work on a factor space representing the exact state S or just an observation O.

- factor spaces: often the state can be coded as a set of (independent) parameters:

  Example: position of the agent + state variables of the environment, Stockmarket: recent course development for all traded stocks, …
- Observation spaces: a set of parameters giving us hints about the state.

  Examples: video buffer of a camera, sensor data, player view in a video game,.

$\Rightarrow$ Since $f(x(s),a, \theta)$ is an approximation function works for both settings ($f(x(s),a, \theta)$ can learn to consider belief states)

$\Rightarrow$ **Caution**: Make sure that $x(s)$ is Markov !!!

Mean Squared Value Error

Regardless of how we built our approximation function $f(S,\theta)$, we need a measure for the quality of an approximation:

$$\overline{VE}^{\pi}(\theta) = \mathbb{E}_{\pi,s\sim\mu}[(U_{\pi}(S) - f(S,\theta))^2]$$
$$= \sum_{s\in S} \mu(S)\,(U_{\pi}(S) - f(S,\theta))^2$$

where μ is the importance distribution over the state descriptions with $\sum_{s\in S}\mu(S)$.

For example, we can take $\mu(S)$ as the likelihood of being in state s when following π.

Common types of function approximators

- Generally any regression/prediction function can be used (usually we will require a continuous return to model the Utility)
- Common methods:
  - Linear predictors
  - Neural networks
  - Decision trees
  - Regression with Fourier/Wavelet bases
  - ..
- However: Reinforcement learning is tricky because:
  - experience is non-stationary (e.g. the label Q(S,A) might change when using TD learning)
  - experience is usually non-iid the observation from a single episode is usually highly correlated

Value Function Approx. with SGD

Goal: Given policy π and $U_\pi(S)$ find θ minimizing a loss function $L^\pi(\theta)$.

Note: We won't have $U_\pi(S)$ but only R(S) later on.

For example, consider $L_{X,Y}(\theta)$ is mean square loss:

$$L^\pi(\theta) = \mathbb{E}_\pi[(U_\pi(S) - f(S,\theta))^2]$$

Computing the gradient we get

$$\Delta\theta = -\frac{1}{2}\alpha\nabla_\theta L^\pi(\theta) = \alpha\mathbb{E}_\pi\left[\left(U_\pi(S) - f(S,\theta)\right)\nabla_\theta L(S,\theta)\right]$$

- With SGD we sample the gradient:

$$\Delta\theta = \alpha\left(U_\pi(S) - f(S,\theta)\right)\nabla_\theta L(S,\theta)$$

- the expected update is equal to the full gradient update

Linear Prediction Functions

A simple function approximation might be linear.

- Linear Functions over $x(S) \in \mathbb{R}^d$ where θ is a weight vector w:

$$f(x(S), \mathrm{W}) = \mathrm{x(S)}^{\mathrm{T}}\mathrm{W} = \sum_{j=1}^{n} x(S)_j^T w_j$$

- Loss function:

$$L(W) = E[(U(s) - \mathrm{x(s)}^{\mathrm{T}}\mathrm{W})^2]$$

- Stochastic Gradient Descent on L(w):

$$\nabla W\ f(x(s), \mathrm{W}) = \mathrm{x(s)}$$

$$-\frac{1}{2}\nabla L(\theta) = \big(U(s) - f(x(s), \theta)\big)x(\mathrm{s})$$

$$\Delta\theta = \alpha\big(U(s) - f(x(s), \theta)\big)x(\mathrm{s})$$

$$\text{update} = step - size \times prediction\ error\ \times feature\ vector$$

Table Lookup Features

- Table lookups can be considered as a special case of linear value function approximation
- Use a lookup table of the of the following form:

$$x^{table}(S) = \begin{pmatrix} 1(S = s_1) \\ \vdots \\ 1(S = s_n) \end{pmatrix}$$

- Parameter vector w gives us the value of each state:

$$f(x(S), w) = \begin{pmatrix} 1(S = s_1) \\ \vdots \\ 1(S = s_n) \end{pmatrix}^T \cdot \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix}$$

Incremental Prediction algorithms

- In practice we do not have the utility $U_\pi(S)$ but only R(S)

$\Rightarrow$ We have to employ a target for $U_\pi(S)$ as in the last lecture

Prediction based on value function approximation:

- For MC, the target is the complete return G_t

$$\Delta\theta = \alpha\big(G_t - f(x(S_t),\theta)\big)\nabla_\theta f(x(S_t),\theta)$$

- For TD, the target is the TD target $R_{t+1} + \gamma f(x(S_{t+1}),\theta)$

$$\Delta\theta = \alpha\big(R_{t+1} + \gamma f(x(S_{t+1}),\theta) - f(x(S_t),\theta)\big)\nabla_\theta f(x(S_t),\theta)$$

- For TD(λ), the target is the λ-return G_t^λ

$$\Delta\theta = \alpha\left(G_t^\lambda - f(x(S_t),\theta)\right)\nabla_\theta f(x(S_t),\theta)$$

Caution: For TD and TD(λ) the target depends on θ

$\Rightarrow$ TD and TD(λ) are semi-gradient methods because the gradient is only computed w.r.t. $f(x(S_t),\theta)$, but the for the target functions.

MC with value function approximation

- Return G_t is an unbiased, noisy sample of true value $U(S)$
- Applying supervised learning to known experience is viable:

  $(x(S_1), G_1)$, $(x(S_2), G_2)$,...$(x(S_T), G_T)$

- For example, linear Monte-Carlo policy evaluation:

$$\begin{aligned}\Delta\theta &= \alpha\big(G_t - f(x(S_t),\theta)\big)\nabla_\theta f(x(S_t),\theta)\\ &= \alpha\big(G_t - f(x(S_t),\theta)\big)\cdot x(S_t)\end{aligned}$$

- Monte-Carlo evaluation converges to a local optimum
- Even when using non-linear value function approximation

TD with value function approximation

- The TD-target is a biased sample sample of true value $U(S)$
- Applying supervised learning is still possible but training data looks like:

$$\big(x(S_1), R_1 + \gamma f(x(S_2),\theta)\big),\ \big(x(S_1), R_2 + \gamma f(x(S_3),\theta)\big), \ldots (x(S_{T-1}), R_T)$$

- For example, linear TD(0) policy evaluation:

$$\begin{aligned}\Delta\theta &= \alpha\big(R_t + \gamma f(x(S_{t+1}),\theta) - f(x(S_t),\theta)\big)\nabla_\theta f(x(S_t),\theta)\\ &= \alpha\big(R_t + \gamma f(x(S_{t+1}),\theta) - f(x(S_t),\theta)\big)\cdot x(S_t)\\ &= \alpha\delta \cdot x(S_t)\end{aligned}$$

- Linear TD(0) converges (close) to global optimum

TD(λ) with value function approximation

- The λ-return is also a biased sample sample of true value $U(S)$
- Applying supervised learning is to training data of the form:

$\left(x(S_1), G_1^{\lambda}\right), \left(x(S_2), G_2^{\lambda}\right), \ldots, \left(x(S_{T-1}), G_{T-1}^{\lambda}\right)$

- Forward view of linear TD(λ):

$$\Delta\theta = \alpha\left(G_1^{\lambda} - f(x(S_t), \theta)\right)\nabla_{\theta} f(x(S_t), \theta)$$
$$= \alpha\left(G_1^{\lambda} - f(x(S_t), \theta)\right) \cdot x(S_t)$$

- Backward view of linear TD(λ):

$$\delta_t = R_{t+1} + \gamma f(x(S_{t+1}), \theta) - f(x(S_t), \theta)$$
$$E_t = \gamma\lambda E_{t-1} + x(S_t)$$
$$= \alpha\delta \cdot E_t$$

Convergence of Prediction Methods

On/Off policy	Algorithm	Table Lookup	Linear	Non-Linear
On-Policy	MC	✓	✓	✓
On-Policy	TD	✓	✓	✗
On-Policy	TD(λ)	✓	✓	✗
Off-Policy	MC	✓	✓	✓
Off-Policy	TS	✓	✗	✗
Off-Policy	TD(λ)	✓	✗	✗

Control and Value Function Approximation

- To apply policy iteration, we again have to switch to state-value functions Q(S,A)

- Basic idea for on-policy learning:
  - approximate q_π with a function q(x(S),a,θ):
    $\hat{q}(S, A, \theta) \approx q_\pi(S, A)$
  - employ $\varepsilon - greedy$ policy improvement

Caution:

- It is not necessary to approximate $q_\pi(S, A)$ very accurately. Instead, we take a step into improving $\hat{q}(S, A, \theta)$ and then adjust the policy.
- Using function approximation is not guaranteed to converge against $q_\pi(S, A)$. Since $\hat{q}(S, A, \theta)$ is a regression function it is not guaranteed that the model can describe the real $q_\pi(S, A)$ for all (S,A).

Control and Value Function Approximation

To learn a reasonable close $\hat{q}(S, A, \theta)$, we can:

- Minimize the mean square error between the approximation $\hat{q}(S, A, \theta)$ and the true action value $q_\pi(S, A)$:

$$L(\theta) = \mathbb{E}_{\pi, s\sim\mu}\left[\left(q_\pi(s, a) - \hat{q}(s, a, \theta)\right)^2\right]$$

- Optimization via SGD:

$$-\frac{1}{2}\nabla_\theta L(\theta) = \left(q_\pi(S, A) - \hat{q}(S, A, \theta)\right)\nabla_\theta \hat{q}(S, A, \theta)$$

$$\Delta\theta = \alpha\left(q_\pi(S, A) - \hat{q}(S, A, \theta)\right)\nabla_\theta \hat{q}(S, A, \theta)$$

Control with Linear Value Functions

- State-action are modelled as a feature vector:

$$x(S,A) = \begin{pmatrix} x_1(S,A) \\ \vdots \\ x_n(S,A) \end{pmatrix}$$

- Represent action-value function by linear combination of features

$$\hat{q}(S\ ,A,w) = \sum_{j=1}^{n} x_j(S,A)w_j$$

- With the SGD update:

$$-\frac{1}{2}\nabla_w L(w) = \big(q_\pi(S,A) - (x(S,A)^T w)\big)\nabla_w (x(S,A)^T w\ )$$
$$= \big(q_\pi(S,A) - (x(S,A)^T w)\big)w$$

$$\Delta w = \alpha\big(q_\pi(S,A) - \hat{q}(S,A,w)\big)x(S,A)$$

Incremental Control Algorithms

similar to prediction but substitute $q_\pi(S, A)$:

- For MC, the target is the complete return G_t

$$\Delta\theta = \alpha\big(G_t - \hat{q}(S_t, A_t, \theta)\big)\nabla_\theta \hat{q}(S_t, A_t, \theta)$$

- For TD, the target is the TD target $R_{t+1} + \gamma\hat{q}(S_t, A_t, \theta)$

$$\Delta\theta = \alpha\big(R_{t+1} + \gamma\hat{q}(S_{t+1}, A_{t+1}, \theta) - \hat{q}(S_t, A_t, \theta)\big)\nabla_\theta \hat{q}(S_t, A_t, \theta)$$

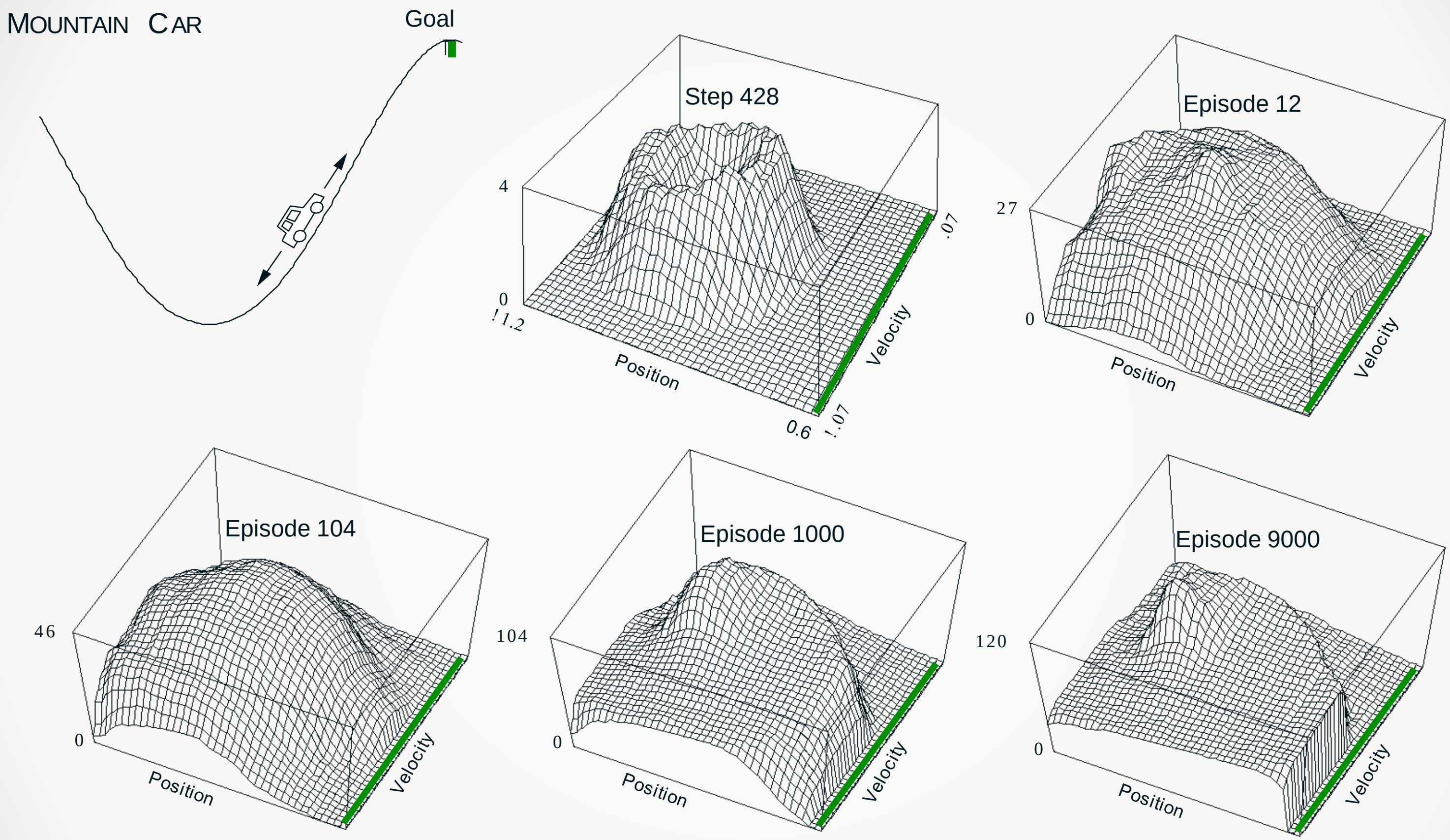
- For TD(λ), the target is the action-value λ-return q_t^λ:
  - Forward view

$$\Delta\theta = \alpha\left(q_t^\lambda - \hat{q}(S_t, A_t, \theta)\right)\nabla_\theta \hat{q}(S_t, A_t, \theta)$$

  - Backward view:

$$\delta_t = R_{t+1} + \gamma\hat{q}(S_{t+1}, A_{t+1}, \theta) - \hat{q}(S_t, A_t, \theta)$$
$$E_t = \lambda\gamma E_{t-1} + \nabla_\theta \hat{q}(S_t, A_t, \theta)$$
$$\Delta\theta = \alpha\delta_t E_t$$

Example: Mountain Car



R. S. Sutton, A. G. Barto: **Reinforcement Learning: An Introduction (Adaptive Computation and Machine Learning),** The MIT Press; Auflage: 2., 2018, page 245, Fig 10.1

Semi-gradient Sarsa method with tile-coding

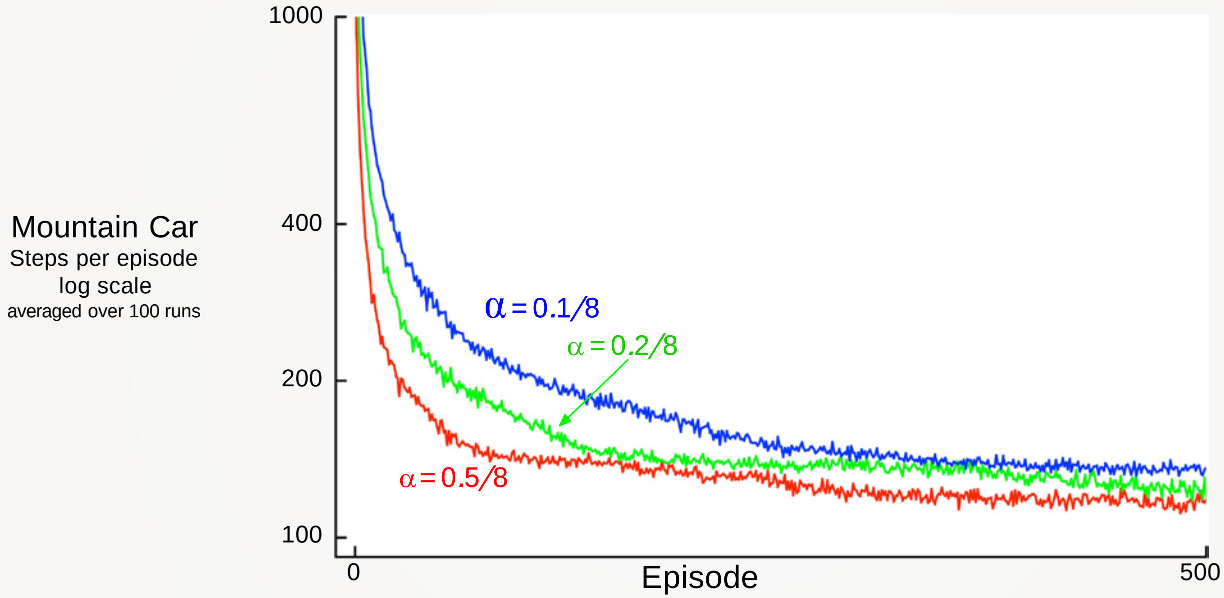


Figure 10.2: Mountain Car learning curves for the semi-gradient Sarsa method with tile-coding function approximation and "-greedy action selection.

Control and Convergence

- Convergence to the minimal error between Q(S,A,θ) and $q_\pi(S, A)$ is problematic.
- Generally convergence is problematic if we employ:
  - Value Function Approximation
  - Bootstraping
  - Off-Policy Learning

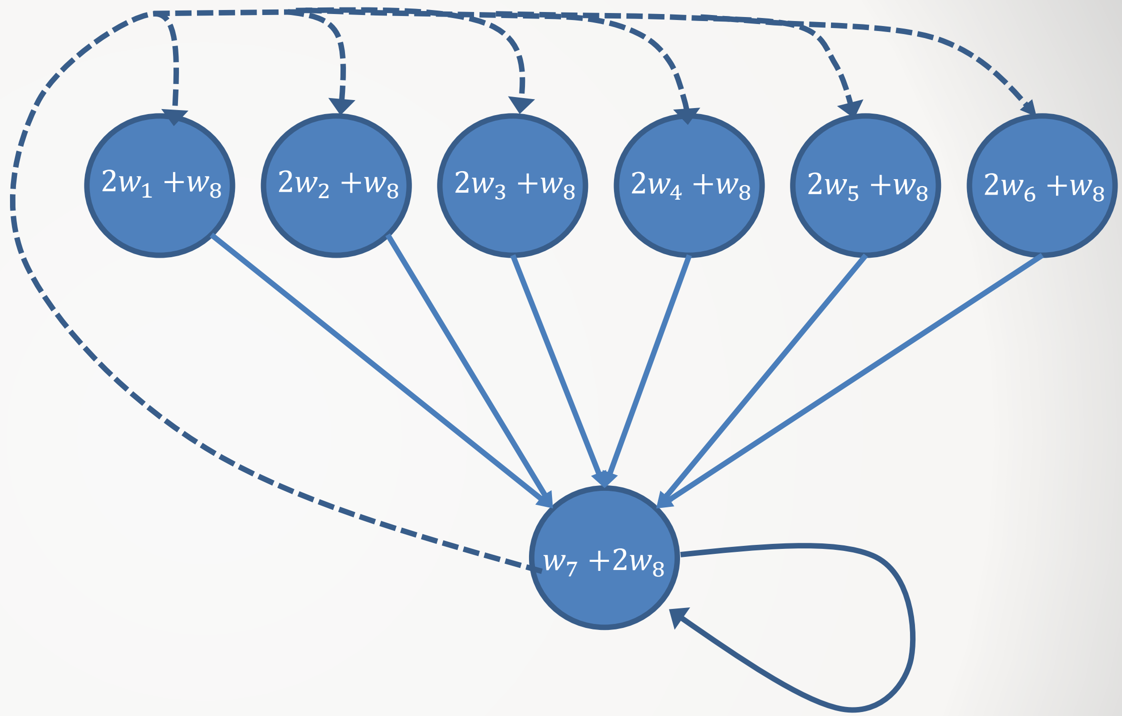
  (Deadly Triad)

=> For these cases, updates might even increase the error.

Baird's Counterexample

- episodic MDP with 7 states and 2 actions:
  - Dashed: go to any of the upper states with 1/6
  - Solid: go to lower state 100 %
- reward is always 0

  $\Rightarrow$ true value functions = 0
- γ=0.999
- behavioural policy b:
  - b$(dashed|\cdot)$ =6/7
  - b$(solid|\cdot)$ =1/7
- target policy:
  - $\pi(solid|\cdot)$ =1.0



feature vectors:

x(1)=$(2,0,0,0,0,0,0,1)^T$

x(2)=$(0,2,0,0,0,0,0,1)^T$

…

x(7)=$(0,0,0,0,0,0,1,2)^T$

Applying semi-gradient TD(0) makes the weights diverge into infinity but switching to on-policy makes the TD(0) converge.

Making gradient methods converge

- TD is not a full GD approach
- Idea: Compute the complete gradient over.
  - Straight-forward the error function is smoothed rather then optimized
  - Gradient TD follows the true gradient of projected Bellman error and therefore does not diverge

On/Off policy	Algorithm	Table Lookup	Linear	Non-Linear
On-Policy	MC	✓	✓	✓
On-Policy	TD	✓	✓	✗
On-Policy	Gradient TD	✓	✓	✓
Off-Policy	MC	✓	✓	✓
Off-Policy	TS	✓	✗	✗
Off-Policy	Gradient TD	✓	✓	✓

Convergence of Control Methods

Algorithm	Table Lookup	Linear	Non-Linear
MC Control	✓	(✓)	✗
Sarsa	✓	(✓)	✗
Q-Learning	✓	✗	✗
Gradient Q-Learning	✓	✓	✗

(✓)= jumps around the near optimal value function

Batch Methods

Utilization of experience is rather bad with GD methods.

$\Rightarrow$ Batch Reinforcement Learning: Find the best fitting value function for the given experience ("training data")

$\Rightarrow$ Using an example only once for making one step might be a waste of experience

Least Squares Prediction

- Given the value function approximation f$(s, \theta) \approx U(S)$
- The experience $\mathcal{D}$ is given by a set of state-value pairs

$$\mathcal{D} = \{\langle s_1, U^\pi(S_1) \rangle, \ldots, \langle s_T, U^\pi(s_T) \rangle\}$$

We want to find the parameters θ to provide the value function approximation f(s, θ) with the best fit on $\mathcal{D}$.

$\Rightarrow$ The least squares method fits the θ to minimize the sum-squared error between f$(s, \theta)$and $U(S)$.

$$LS_{\mathcal{D}}(\theta) = \sum_{t=1}^{T} (U^\pi(s_t) - f(s, \theta))^2$$
$$\cong \mathbb{E}_{\mathcal{D}}[(U^\pi(s_t) - f(s, \theta))^2]$$

SGD with Experience Replay

Given experience $\mathcal{D}$ is given by a set of state-value pairs

$$\mathcal{D} = \{\langle s_1, U^\pi(S_1) \rangle, \ldots, \langle s_T, U^\pi(s_T) \rangle\}$$

Repeat:

- Smple state-value pair from $\mathcal{D}$:

$$\big\langle \langle s, U^\pi(s) \rangle \big\rangle \sim \mathcal{D}$$

- Apply SGD update on the parameters θ:

$$\Delta\theta = \alpha\big(U^\pi(s) - f(s,\theta)\big)\nabla_\theta f(s,\theta)$$

$\Rightarrow$ Converges to least squares solution

$$\theta^\pi = arg \max_\theta LS_\mathcal{D}(\theta)$$

Experience Replay in Deep Q-Networks (DQN)

- DQN applies deep learning to off-policy, non-linear, TD-target reinforcement learning. (danger of instability)
- by using experience replay and fixed Q-targets can be trained in a stable way.
- Experience:
  - observed transitions $(s_t, a_t, r_{t+1}, s_{t+1})$ in replay memory $\mathcal{D}$.
  - By sampling independently from $\mathcal{D}$ episodes are decoupled

- Idea of fixed Q-targets:
  - Q-learning targets in the experience replay are all generated w.r.t. "old", fixed parameters θ^-.
  - Thus, Q-targets are independent from θ in f(s, θ) which are updated

DQN Algorithm

Repeat:

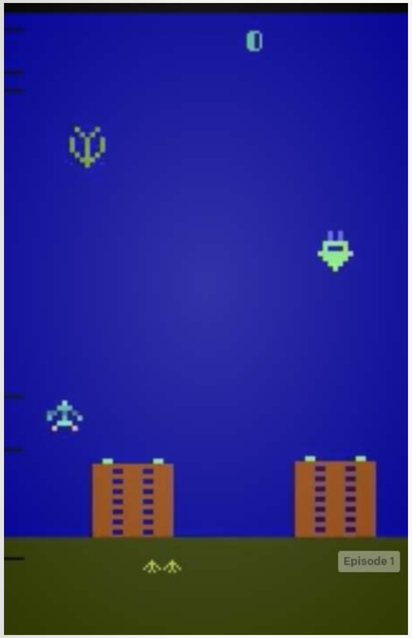
- Take action a_t according to ε-greedy policy
- Store transition $(s_t, a_t, r_{t+1}, s_{t+1})$ in replay buffer $\mathcal{D}$
- Sample random mini-batch of transition (s,a,r,s') from $\mathcal{D}$
- Compute the Q-learning targets w.r.t. fixed θ^{-}
- Optimize MSE between Q-network and Q-learning targets:

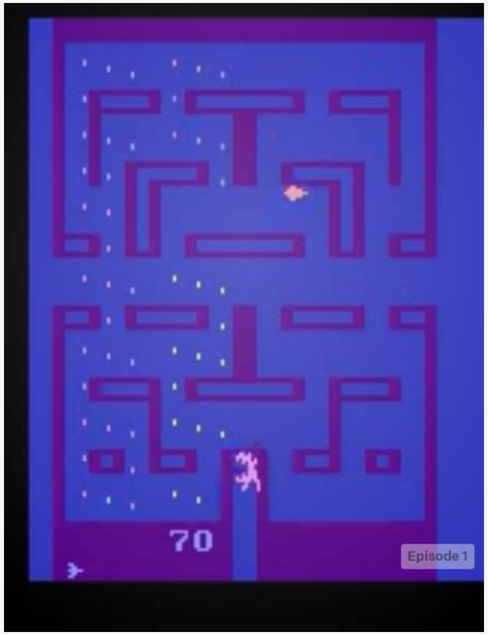
$$\mathcal{L}_i(\theta_i) = \mathbb{E}_{s,a,r,s' \sim \mathcal{D}} \left[\left(r + \gamma \max_{a'} f(s', a'; \theta^{-}) - f(s, a, \theta) \right)^2 \right]$$

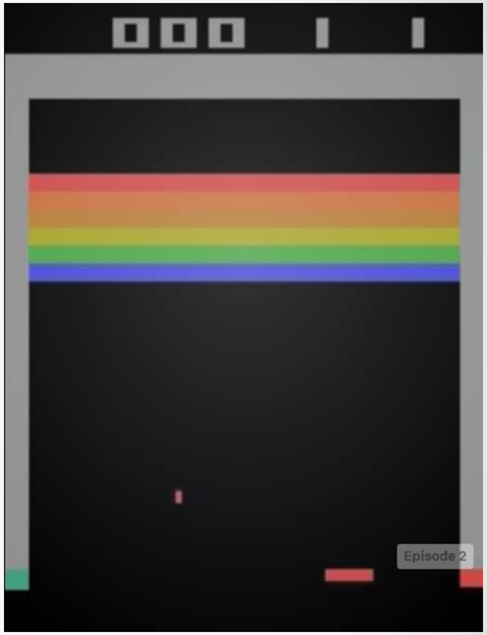
by SGD

Example: DQN on Atari games

https://gym.openai.com/envs/#atari



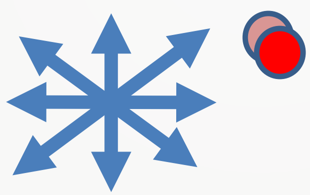




DQN in Atari

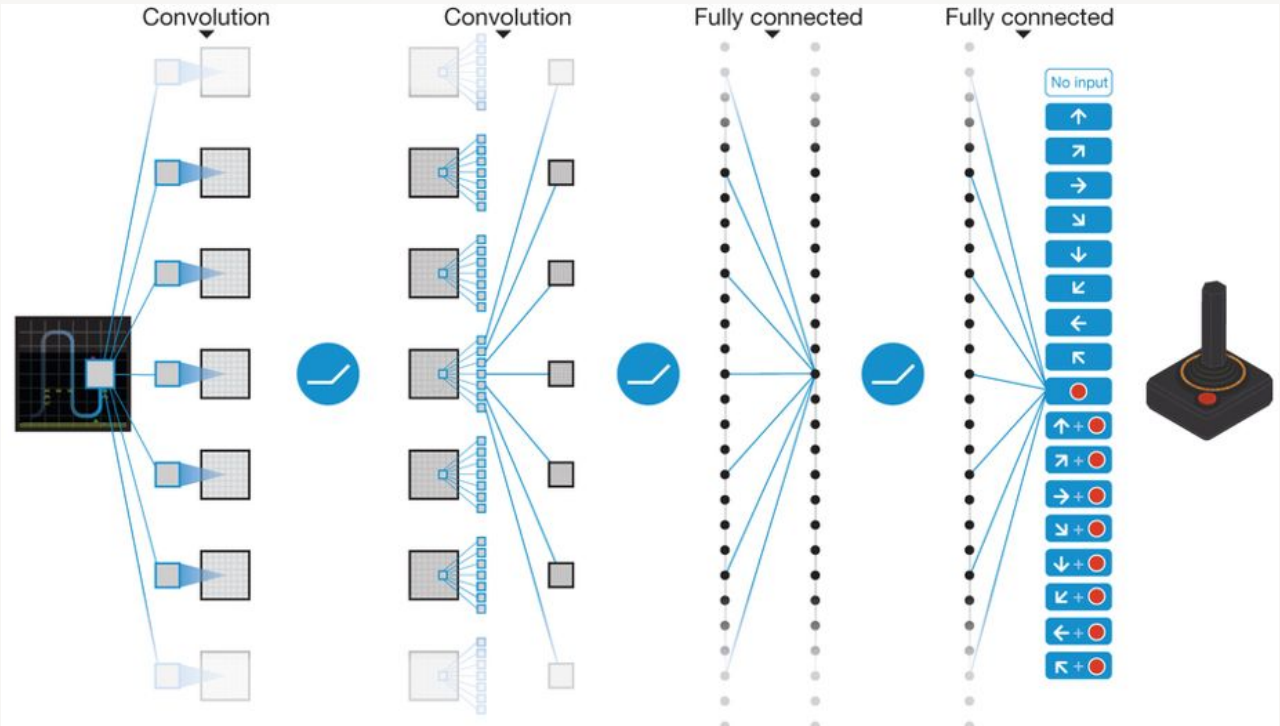
Idea: Use one network architecture to learn multiple computer games on the video buffer as input.

- End-to-end learning of Q(s,a) from pixels s
- Input state s is stack of raw pixels from last 4 frames (a single fram is not Markov!!)
- Actions: 18 Joystik/button combnation (9 directions + 2 button states)



- Reward change in score for the step (most Atari games had general scores constantly rewarding actions)

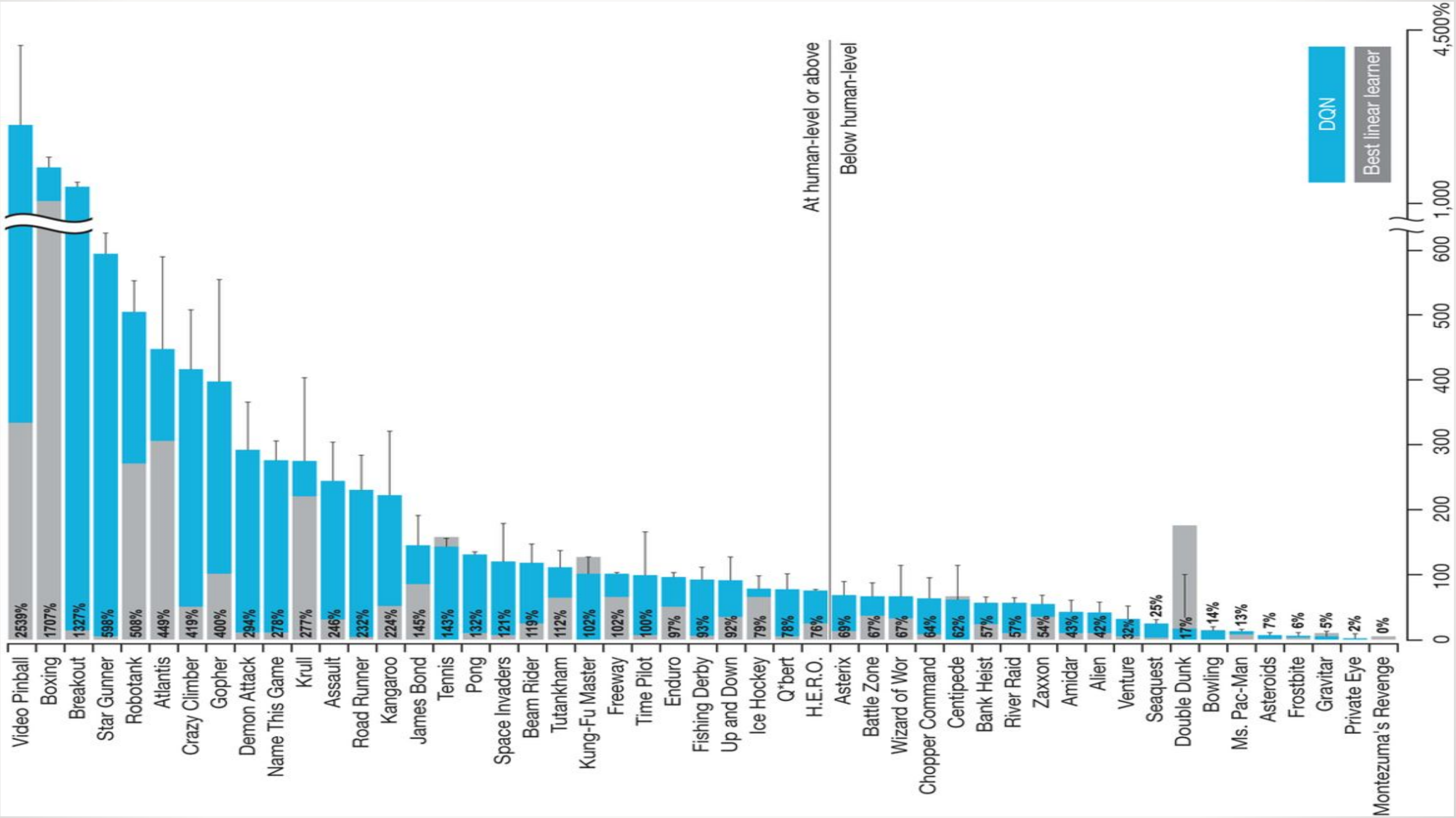
DQN on Atari Games Network Architecture



https://media.nature.com/lw926/nature-assets/nature/journal/v518/n7540/images/nature14236-f1.jpg

Mnih, V., Kavukcuoglu, K., Silver, D., Rusu, A. A., Veness, J., Bellemare, M. G., Graves, A., Riedmiller, M., Fidjeland, A. K., Ostrovski, G., Petersen, S., Beattie, C., Sadik, A., Antonoglou, I., King, H., Kumaran, D., Wierstra, D., Legg, S. & Hassabis, D. (2015). Human-level control through deep reinforcement learning. *Nature*, 518, 529--533.

DQN Results in Atari



Advantages replay buffer and fixed targets

	Replay Fixed-Q	Replay Q-learning	No replay Fixed-Q	No replay Q-learning
Breakout	316.81	240.73	10.16	3.17
Enduro	1006.3	831.25	141.89	29.1
River Raid	7446.62	4102.81	2867.66	1453.02
Seaquest	2894.4	822.55	1003	275.81
Space Invaders	1088.94	826.33	373.22	301.99

Lecture notes D. Silver: Introduction to Reinforcement Learning (http://www0.cs.ucl.ac.uk/staff/d.silver/web/Teaching.html)

Lecture 6: Function Approximation (Slide 41)

Linear Least Squares Prediction

- Experience replay finds the least-squares solutions by sampling and using SGD on mini-batches
- If we use linear value function approximation, we can analytically solve for a solution minimizing $LS_{\mathcal{D}}(\theta)$:

$$LS_{\mathcal{D}}(w) = \sum_{t=1}^{T} (U^{\pi}(s_t) - x_t^T w)^2$$

- For a quadratic loss the derivative is linear and to find a local minimum we have to compute its zero

Linear Least Squares Prediction

- At minimum $LS_{\mathcal{D}}(w) = 0$,i.e., the expected update is zero as well.

$$\mathbb{E}_{\mathcal{D}}[\Delta w] = 0$$

$$\alpha \sum_{t=1}^{T} x(s_t)(U^{\pi}(s_t) - x(s_t)^T w) = 0$$

$$\textstyle\sum_{t=1}^{T} x(s_t) U^{\pi}(s_t) = \sum_{t=1}^{T} x(s_t) x(s_t)^T w$$

$$\mathrm{w} = \left(\sum_{t=1}^{T} x(s_t) x(s_t)^T \right)^{-1} \sum_{t=1}^{T} x(s_t) U^{\pi}(s_t)$$

- For N features, direct solution $O(N^3)$ (matrix inversion)
- Incremental solution time $O(N^2)$ using Sherman-Morrision
- Usability depends on the ratio between the number of features N and the number of samples T

Linear Least Squares Prediction Algorithms

- Again: in practice $U^{\pi}(s_t)$ is yet unknown

$\Rightarrow$ Use noisy or biased samples of $U^{\pi}(s_t)$

- LMSC - Least Squares Monte-Carlo

  $U^{\pi}(s_t) \approx G_t$
- LSTD – Least Squares Temporal Difference Learning

  $U^{\pi}(s_t) \approx R_{t+1} + \gamma f(s_t, w)$
- LSTD(λ) – Least Squares with TD(λ)

  $U^{\pi}(s_t) \approx G_t^{\lambda}$

$\Rightarrow$ For each target we can solve directly for the fixed point of MC/TD/ TD(λ) because the targets are considered as fixed

Linear Least Squares Prediction Algorithms (2)

LMSC: $0 = \alpha \sum_{t=1}^{T} (G_t - x(s_t)^T w) x(s_t)$

$$w = \left(\sum_{t=1}^{T} x(s_t) x(s_t)^T\right)^{-1} \sum_{t=1}^{T} x(s_t) G_t$$

LSTD: $0 = \alpha \sum_{t=1}^{T} (R_{t+1} + \gamma x(s_{t+1})^T w - x(s_t)^T w) x(s_t)$

$$w = \left(\sum_{t=1}^{T} x(s_t)\big(x(s_t) - \gamma x(s_{t+1})\big)^T\right)^{-1} \sum_{t=1}^{T} x(s_t) R_{t+1}$$

LSTD(λ): $0 = \sum_{t=1}^{T} \alpha \delta_t E_t$

$$w = \left(\sum_{t=1}^{T} E_t\big(x(s_t) - \gamma x(s_{t+1})\big)^T\right)^{-1} \sum_{t=1}^{T} E_t R_{t+1}$$

Convergence of Linear Least Square Prediction

On/Off policy	Algorithm	Table Lookup	Linear	Non-Linear
On-Policy	MC	✓	✓	✓
	LSMC	✓	✓	-
	TD	✓	✓	✗
	LSTD	✓	✓	-
Off-Policy	MC	✓	✓	✓
	LSMC	✓	✓	-
	TD	✓	✗	✗
	Gradient TD	✓	✓	-

Least Squares Control Algorithms

- Policy evaluation: Policy evaluation by least squares Q-Learning
- Policy improvement: Greedy policy improvement
- We want to make use of all experience in $\mathcal{D}$ to:
  - Efficienty evaluate the policy
  - Improve the policy

But: The experience is drawn from different policies from various stages of training.

$\Rightarrow$ To evaluate $q_\pi(S, A)$we must learn off-policy

$\Rightarrow$ Use the same idea as on Q-learning:

- Use experience generated by old policy: S_t,A_t,R_{t+1},S_{t+1}~π_{old}
- Consider alternative successor action A′=$\pi_{new}(S_{t+1})$
- Update f(S_t,A_t,w) towards value of alternative action:

$$R_{t+1} + \gamma f(S_{t+1}, A', w)$$

Least Squares Q-Learning

- Given the following Q-learning update

$$\delta = R_{t+1} + \gamma\, f(S_{t+1}, \pi(S_{t+1}), w) - f(S_t, A_t, w)$$
$$\Delta w = \alpha \delta x(S_t, A_t)$$

- LSTDQ algorithm: solve for total update = zero

$$0 = \sum_{t=1}^{T} \alpha\Big(R_{t+1} + \gamma\, f(S_{t+1}, \pi(S_{t+1}), w) - f(S_t, A_t, w)\Big) - f(S_t, A_t, w) x(S_t, A_t)$$

$$w = \left(\sum_{t=1}^{T} x(S_t, A_t)\Big(x(S_t, A_t) - \gamma x\big(S_{t+1}, \pi(S_{t+1})\big)\Big)^{T}\right)^{-1} \sum_{t=1}^{T} x(S_t, A_t)\, R_{t+1}$$

Least Squares Policy Iteration Algorithm

- Pseudocode Policy Iteration using LSTDQ
- Experience $\mathcal{D}$ is re-evaluated with different policies

function LSPI-TD($\mathcal{D}$, π_0)
    $\pi' \leftarrow \pi_0$
    repeat
        $\pi \leftarrow \pi'$
        $Q \leftarrow$ **LSTDQ**(π, D)
        for all $s \in S$ **do**
            $\pi'(s) \leftarrow \arg\max_{a \in A} Q(s, a)$
        end for
    until ($\pi \approx \pi'$)
    return π
End function

Convergence of Control Algorithms

Algorithm	Table Lookup	Linear	Non-Linear
MC Control	✓	(✓)	✗
Sarsa	✓	(✓)	✗
Q-Learning	✓	✗	✗
LSPI	✓	(✓)	-

(✓)= jumps around the near optimal value function

Literature

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